Improving Health Outcomes Through Better Capacity Allocation in a Community-Based Chronic Care Model

Sarang Deo
Indian School of Business, Gachibowli, Hyderabad, India 500032, sarang_deo@isb.edu

Seyed Iravani, Tingting Jiang, Karen Smilowitz
Department of Industrial Engineering and Management Sciences, Northwestern University, Evanston, Illinois 60208
{s-iravani@northwestern.edu, tingting-jiang@northwestern.edu, ksmilowitz@northwestern.edu}

Stephen Samuelson
Frisbie Senior Center, Des Plaines, Illinois 60016, steves@frisbieseniorcenter.org

This paper studies a model of community-based healthcare delivery for a chronic disease. In this setting, patients periodically visit the healthcare delivery system, which influences their disease progression and consequently their health outcomes. We investigate how the provider can maximize community-level health outcomes through better operational decisions pertaining to capacity allocation across different patients. To do so, we develop an integrated capacity allocation model that incorporates clinical (disease progression) and operational (capacity constraint) aspects. Specifically, we model the provider’s problem as a finite horizon stochastic dynamic program, where the provider decides which patients to schedule at the beginning of each period. Therapy is provided to scheduled patients, which may improve their health states. Patients that are not seen follow their natural disease progression. We derive a quantitative measure for comparison of patients’ health states and use it to design an easy-to-implement myopic heuristic that is provably optimal in special cases of the problem. We employ the myopic heuristic in a more general setting and test its performance using operational and clinical data obtained from Mobile C.A.R.E. Foundation, a community-based provider of pediatric asthma care in Chicago. Our extensive computational experiments suggest that the myopic heuristic can improve the health gains at the community level by up to 15% over the current policy. The benefit is driven by the ability of our myopic heuristic to alter the duration between visits for patients with different health states depending on the tightness of the capacity and the health states of the entire patient population.

Subject classifications: capacity allocation; chronic disease; mobile care; disease progression; appointment scheduling.
Area of review: Policy Modeling and Public Sector OR.
History: Received October 2010; revisions received October 2011, November 2012, March 2013; accepted June 2013.

1. Introduction

There has been a recent increase in the application of operations research models to healthcare delivery (see Branderup et al. 2004). Most models can be broadly partitioned in two categories based on their focus: (i) improvement in efficiency of the healthcare delivery system (reduce cost, increase revenue, reduce waiting time) and (ii) improvement in effectiveness of clinical decisions for individual patients (optimal time to initiate treatment, transplant an organ). In this paper, we consider a novel setting of community-based healthcare delivery for chronic diseases, where it is necessary to integrate these two approaches, and investigate how operational decisions can improve population-level health outcomes.

Community-based chronic care is growing in North America, fueled by an aging population and growing disparities in access to care. These factors necessitate outreach to disadvantaged population groups in their communities rather than waiting for them to access healthcare in conventional settings. For example, Project Dulce run by Scripps Health provides diabetes care and management to thousands of ethnically diverse and low-income patients in San Diego County (http://www.scripps.org/services/diabetes/project-dulce). The Diabetes Integration Project in Manitoba provides care through mobile teams (http://www.diabetesintegrationproject.ca/). Similar programs have been initiated to target homeless populations (Post 2007).

Several features of these settings render conventional healthcare planning and scheduling models inapplicable because of the interaction of operational decisions and health outcomes. First, objective functions in most healthcare planning and scheduling models relate to efficiency or patient satisfaction, e.g., cost minimization, revenue maximization, waiting time minimization (Gupta and Denton 2008), and the impact on health outcomes is not modeled explicitly. In community-based care, the most natural objective is to maximize the aggregate health outcomes of the target population subject to resource constraints due to the nonprofit and/or public model of delivery. Second, primary care or surgical procedures, which are the focus
of most operational models, involve episodic access of the healthcare system by patients. However, chronic care requires repeated interaction with patients whose disease states evolve over time as a function of prior care. Time between consecutive visits and health state at the prior visit have a direct impact on disease progression.

In this paper, we propose an integrated capacity allocation approach that combines clinical (disease progression) and operational (scarce capacity) elements. Specifically, we formulate a finite horizon stochastic dynamic program for allocating limited appointment slots among patients with different health states to maximize aggregate health benefits measured in Quality Adjusted Life Years (QALYs) gained. Equivalently, the problem could be interpreted as deciding the optimal duration between visits for patients with varying degrees of illness. We derive the structural properties of the optimal capacity allocation policy when the matrices governing the transition of patients across different health states satisfy certain properties. We also develop an easily implementable heuristic that is optimal under these conditions and gives near-optimal performance when these conditions are not satisfied.

We calibrate our model and apply our results to data collected from Mobile C.A.R.E. Foundation (MCF), a non-profit organization that provides school-based asthma care for children in Chicago. Given a roster of active schools and a schedule of visits to these schools, MCF staff determines the capacity allocation at each school through daily patient schedules. The schedules are based on the medically recommended treatment durations for the patients that are modified based on the available capacity at each school. We conduct extensive computational experiments using clinical and operational data from MCF to test the performance of our myopic heuristic. For small values of capacity, where the optimal policy can be calculated through brute force enumeration, it provides near-optimal performance (with an average optimality gap of 0.4% over all instances). For large capacity values, where optimal capacity calculation is not possible, it provides significant improvement (up to 15%) in health outcomes over current practice, which is very close to that of more complex approaches such as the Whittle’s index.

Our paper makes several contributions to the healthcare operations literature. First, we consider a novel setting of community-based healthcare delivery for chronic diseases in a nonprofit setting, which necessitates the integration of clinical (disease progression) and operational (capacity constraint) decisions into a single decision model to maximize community-level health outcomes. Second, we propose a quantitative method of ordering patients with different health states in the absence of perfect observability because all patients are not seen in each period due to capacity constraints. Third, we use this quantitative characterization to devise a simple and implementable patient scheduling approach that provides near-optimal performance. Fourth, we calibrate the model with data from a mobile provider of childhood asthma care and demonstrate that community-level health outcomes can be improved significantly by making more effective operational decisions pertaining to capacity allocation without changing therapeutic decisions. Fifth, we develop a simple heuristic for the capacity allocation problem that is easily implementable in a spreadsheet software package and whose performance is very close to more sophisticated algorithms. Thus, our work highlights the significant potential of improving access to community-based chronic care programs by applying operations research methods to improve capacity allocation decisions.

The remainder of the paper is organized as follows. The clinical context of childhood asthma and related community-based programs are described in §2. Relevant streams of operational and healthcare literature are discussed in §3. The components of the capacity allocation model are developed in §4. The structural properties of the optimal policy is illustrated in §5. In §6 we develop an easy to implement and efficient heuristic. Operational decision making at MCF is described in detail in §7, followed by model calibration. Results of our numerical study evaluating the improvement over current practice and associated policy implications are discussed in §8. Concluding remarks and potential avenues for future research are presented in §9. Tables containing the key empirical findings and proofs of theoretical results are included in the technical appendix (available as supplemental material at http://dx.doi.org/10.1287/opre.2013.1214).

2. Childhood Asthma and Community-Based Programs

We focus on community-based care for childhood asthma because of the magnitude of disease burden, recent growth of such delivery models, lack of analytical models to inform operations decisions, and MCF’s willingness to collaboratively develop and implement these models.

Childhood asthma, a chronic respiratory disease characterized by periods of difficulty in breathing, affects about 6.5 million children in the United States. Childhood asthma places severe burden on families and society at large through healthcare resource use because asthma management requires regular monitoring by healthcare providers even during asymptomatic periods. Absence of appropriate care can lead to asthma attacks. In 2004, such attacks resulted in 12.8 million missed school days, 750,000 emergency department visits, and 198,000 hospitalizations in the United States (Akinbami 2006). Severe disparities exist in the prevalence of childhood asthma (higher), the resultant healthcare utilization (lower), and mortality (higher) among many ethnic minorities such as Native American, African American, and Puerto Rican compared to white children (Marder et al. 1992). These disparities have been attributed to lack of health insurance coverage and awareness among parents and caregivers. Studies have found that children in
inner city neighborhoods have higher prevalence of asthma likely due to factors such as allergens, tobacco smoke, pollution and poorly ventilated houses (Kitch et al. 2000), and family income levels (Gupta et al. 2008).

Community-based programs have been shown to reduce these disparities by improving access to health services for the underserved populations, including those in Chicago, Los Angeles, Baltimore, and St. Louis. These programs typically consist of a mobile clinic with the following common characteristics: (i) a school-based model, (ii) patient recruitment with involvement of parents and school nurses, and (iii) continuous patient follow-up using appointment scheduling and periodic school visits. Recent studies demonstrate the success of such programs in controlling asthma (Jones et al. 2007). Although a return on investment calculator for mobile clinics has been developed (Oriol et al. 2009), there are no mathematical models to explicitly account for the impact of operational decisions on population-level health outcomes. In this paper, we aim to fill this gap by investigating how improved capacity allocation can improve health outcomes for a population of asthmatic children, and we apply our findings to MCF as a case study.

3. Literature Review

We discuss existing work in four streams of relevant literature. The first two streams focus, separately, on the two aspects of healthcare delivery—appointment scheduling and medical decision making—that we combine. The third stream, literature on restless multiarmed bandit problems, focuses on developing solution methods for problems with a structure similar to ours. The fourth stream comprises models of machine maintenance and repair that have structural similarity to ours.

Appointment scheduling literature. Gupta and Denton (2008) review healthcare appointment scheduling challenges in three common systems, namely, primary care (outpatient facilities), specialty clinics, and elective surgeries (inpatient facilities). Models of these appointment systems consider patients that arrive punctually for their scheduled appointment time to a single physician and experience stochastic service times that are identically and independently distributed. The key decisions involved are length of the appointment slot (Cayirli and Veral 2006), appointment times (Robinson and Chen 2003), and the number of appointment requests to accept including overbooking (Gupta and Wang 2008). The objective is to minimize the total cost of direct patient waiting (inside the clinic) and physician idleness and/or overtime (Denton and Gupta 2003). Recent papers analyze open access systems (Murray and Tantau 1999) where some slots are reserved for same-day arrivals (Liu et al. 2010, Robinson and Chen 2010). The key trade-off here is between reduction of indirect waiting time (between a request for appointment and the actual appointment) and a possible increase in direct waiting time and physician overtime. Our model differs from this literature in its decision variable and its objective. The key decision in our model is the selection of patients to schedule in each period and our objective, motivated by the nonprofit setting, is to maximize aggregate QALYs for the patient population.

Our work is distinct because of our focus on chronic care whereby patients return for follow-up visits instead of unidentifiable appointment requests as in the above papers. Gupta and Denton (2008) highlight the paucity of planning and scheduling models for chronic conditions, which largely remains unaddressed with a few exceptions. Lee and Zenios (2009) develop a semiclosed migration network to model patient flow between different compartments of a chronic care system (end stage renal disease) with the objective of maximizing the revenue. Kucukyazici et al. (2011) also use a patient flow approach with an epidemiological model to analyze the performance of different patient flow configurations for chronic care (stroke) using simulation. Our work differs from these papers in two respects. First, we model one component of the chronic care system (the clinic) in greater detail and exclude other components such as emergency room and tertiary care hospital. Second, we explicitly model the dependence of the capacity allocation decision in this component on the health states of the entire patient population, which is missing in these papers.

Medical decision-making literature. Several operations research models have been proposed recently to optimize timing of clinical decisions such as liver transplants (Alagoz et al. 2004) and initiation of HIV therapy (Shechter et al. 2008). Although these models employ disease progression models, they typically consider individual patient outcomes in contrast to our focus on population health outcomes. Some models of organ transplant consider population-level dynamics (e.g., Zenios et al. 2000) but differ from our model because of the one-time nature of their intervention as opposed to periodic follow-up in our case. There is a vast literature on cost effectiveness of medical interventions (Sonnemberg and Beck 1993), including for asthma (Paltiel et al. 2006), that analyzes disease progression at the population level. However, these models consider theoretical cohorts of patients, do not model capacity constraints in the healthcare system, and do not optimize over decisions (Brennan et al. 2006). In contrast, we explicitly model the healthcare delivery system and develop models to generate qualitative insights and provide decision support on the ground.

Restless multiarmed bandit problem literature. Our model belongs to a general class of dynamic resource allocation problems: restless multiarmed bandit (RMAB) problem (Whittle 1988). In each period, the decision maker chooses the number of arms to activate (the number of projects to pursue), subject to a capacity constraint, with the objective of maximizing the total discounted return over a long time horizon. Return from each project depends on its current state (active or passive). The state of each project
evolves stochastically; the transition probabilities depend on whether the project was active or passive in that period. The RMAB problem is proven to be intractable in general (Papadimitriou and Tsitsiklis 1994), and hence the majority of work in this stream has focused on developing heuristics. One of the most widely used heuristics is Whittle’s index that is based on Lagrangian relaxation of the capacity constraint linking the projects. The index corresponds to a (state-dependent) subsidy that makes the decision maker indifferent between keeping a project passive and activating it. The projects are ranked based on the index and the ones with the highest index, subject to the capacity constraint, are chosen to be active.

Weber and Weiss (1990) prove that Whittle’s index is asymptotically optimal when the ratio of the active number of bands to the total number is fixed. Most papers (e.g., Ansell et al. 2003; Glazebrook et al. 2005, 2006) focus on proving indexability before proceeding to compute Whittle’s index and numerically solving the problem. In our model, we prove that a much simpler myopic policy is optimal in special settings of the problem and use the policy to devise a heuristic for more general settings. The performance of this heuristic is comparable to Whittle’s index, but the structure is much simpler, thus making it more attractive from a practical perspective. Liu and Zhao (2009) prove the optimality of myopic policy for a special case of our model. In their model, action in each period corresponds to only inspection of active arms. In our model, action corresponds to inspection of active arms combined with treatment, which complicates the state transition dynamics.

Machine maintenance and repair literature. Our stochastic dynamic program is related to models of machine repair and maintenance with multiple units (e.g., Wang 2002). At an abstract level, our model is most closely related to Glazebrook et al. (2005), which studies the optimal allocation of repairmen to machines that deteriorate under usage. However, Glazebrook et al. (2006) assume that the states of machines are known at every decision epoch. In our model, since a patient’s health state is not observable without a diagnosis during the visit and every patient is not seen each period, we assume that the decision maker only knows the probability distribution over health states. The dynamics of the Markov decision process (MDP) problem in our model is similar to the dynamics of single machine maintenance problems. For example, Pham and Wang (1996) characterize a machine’s status as a binary variable, i.e., good or bad and consider “imperfect repair,” i.e., the state of the machine after repair might not always return to the best possible one. Rosenfield (1976) studies the optimal policy for deteriorating processes with imperfect information that is governed by a discrete time Markov process. Our model differs from this literature by extending the unconstrained setting to a resource constrained setting. In the dynamics of the MDP problem, our model also differs from machine maintenance models with imperfect information (Valdez-Flores and Feldman 1989) in that inspection (diagnosis) and repair (treatment) are inseparable in our setting.

4. Model

We formulate the capacity allocation problem as a discrete-time, finite-horizon, discounted MDP model with the objective of maximizing the QALYs for the entire patient cohort. We propose an integrated approach to capacity allocation that combines both operational and clinical decisions. In contrast, the current policy of capacity allocation at MCF, as we describes in §7.2, makes operational and clinical decisions in a decoupled manner in two stages.

Assuming a fixed schedule of equally spaced visits to a school, the sequence of events in a period in our model are as follows:

- Payoff (in terms of QALY values) of the previous period’s decision is collected.
- Capacity allocation decisions for the current period are made.
- Appointments occur for patients scheduled in the current period.
- True health state is diagnosed.
- Treatment is applied and takes effect immediately.
- Natural disease progression occurs for all patients.

The description of the model is divided into two parts: §§4.1 through 4.3 present a discrete-time Markov chain for individual patient disease progression, and §4.4 presents a constrained Markov decision process for allocating capacity among all patients.

4.1. Individual Patient Model: Disease Progression and Treatment

Consistent with the disease progression models of asthma in the medical literature (Paltiel et al. 2006, Shahani et al. 1994), we consider a homogeneous patient population of $I$ patients, whose disease progression is governed by a Markov process over discrete health states $0, 1, \ldots, K - 1$, where $0$ represents the best and $K - 1$ represents the worst health states, respectively. We consider these patients over a finite horizon of $T$ periods.

At the beginning of period $t \in \{1, 2, \ldots, T\}$, patient $i$’s state is given by a tuple, $\bar{s}_{i,t} = (h_{i,t}, n_{i,t})$, where $h_{i,t} \in \{0, 1, \ldots, K - 1\}$ represents the health state diagnosed at the last appointment and $n_{i,t} \in \{1, 2, \ldots, T - 1\}$ represents the time since last appointment measured by number of periods.

The natural disease progression, without any medical intervention, is characterized by a per-period transition matrix $\mathcal{P}$, where $\mathcal{P}_{lk}$, the $(l,k)$th element of matrix $\mathcal{P}$, is the probability that a patient who is in health state $l$ will be in health state $k$ after one period, $\sum_{k = 0}^{K-1} \mathcal{P}_{lk} = 1, \forall l \in \{0, 1, \ldots, K - 1\}$ and $\mathcal{P}_{lk} \geq 0, \forall l, k \in \{0, 1, \ldots, K - 1\}$. In the presence of medical intervention, the state transition is governed by both treatment and natural disease progression, which we explain next.

A typical clinical appointment for chronic care involves diagnosis of patient’s current health state and adjustment of future treatment plan. Treatment for chronic conditions...
consists of a clinical component (modifying prescribed drugs and/or their dosages) and a behavioral component (counseling/education of patients and caregivers). Clearly, the effect of treatment is strongest immediately following the appointment but wanes over time until the next appointment. However, explicitly modeling this temporal effect introduces nonstationarity and severely hampers the analytical tractability of our model. To simplify this complex situation, we assume that the treatment effect occurs immediately after the appointment, improving the patient’s current health state. This is modeled by a lower triangular treatment matrix \( \mathbf{c} \), where the \((l, k)\)th element in matrix \( \mathbf{c} \) is denoted by \( c_{lk} \) where \( \sum_{k=0}^{\infty} c_{lk} = 1, \forall l \in \{0, 1, \ldots, K-1\} \) and \( c_{lk} \geq 0, \forall l, k \in \{0, 1, \ldots, K-1\} \). After the treatment effect occurs, the patient’s disease progression is again governed by \( \mathcal{P} \). We assume the disease progression process and treatment process are independent of each other. The effective state transition of patients can be interpreted as a Markov chain with two transition rates that depend on the time since the last visit. The patient transition matrix is \( \mathbf{c} \mathcal{P} \) for the first period after the visit and \( \mathcal{P} \) for all subsequent periods until the next appointment. Note that although the isolated effect of treatment is to only improve patient health, the combined effect of \( \mathbf{c} \) and \( \mathcal{P} \) between two visits might be such that a patient under treatment can transition to a worse health state before the next visit.

### 4.2. Information Vector

When making capacity allocation decisions, the current health states of the patients might not be known with certainty. We use random variable \( x_{i,t} \) to denote patient \( i \)’s true health state at the beginning of period \( t \) and \( x_{i,1} \in \{0, 1, \ldots, K-1\} \) as its realizations. We assume that patients’ health states are known at the beginning of period 1; i.e., \( x_{i,1} \) is known with certainty for all \( i \). When \( t > 1 \), for patient \( t \) with state \( \tilde{s}_{i,t} = (h_{i,t}, n_{i,t}) \), the distribution of \( x_{i,t} \) is given by \( \mathbf{\pi}_{i,t} = \mathbf{c}^{h_{i,t}} \mathcal{P}^{n_{i,t}} \), where \( \mathbf{c}^{h_{i,t}} \) is a row vector of \( K \) zeros with a one in its \((k+1)\)th element. We refer to \( \mathbf{\pi}_{i,t} \) as the information vector of patient \( i \) at the beginning of period \( t \), representing the healthcare provider’s belief about patient \( i \)’s true health state at the beginning of period \( t \) before an allocation decision is made and before the patient is seen. Note that for an information vector with \( K \) elements, the first \( K-1 \) elements are sufficient to define the entire vector since all elements sum to 1.

Given patient \( i \)’s information vector \( \mathbf{\pi}_{i,t} \) in period \( t \), patient \( i \)’s information vector at the beginning of period \( t + 1 \) is a random vector given by

\[
\mathbf{\pi}_{i,t+1} = \begin{cases} 
\mathbf{\pi}_{i,t} \mathcal{P}, & \text{with probability } (\mathbf{\pi}_{i,t})_k, \\
\mathbf{\pi}_{i,t} \mathcal{P}, & \text{if patient } i \text{ is scheduled in period } t, \\
\mathbf{\pi}_{i,t} \mathcal{P}, & \text{otherwise}.
\end{cases}
\]

Here, \((\mathbf{\pi}_{i,t})_k\) is the \((k+1)\)th element in vector \( \mathbf{\pi}_{i,t} \), representing the probability that patient \( i \) is in health state \( k \) at the beginning of period \( t \) before a capacity allocation decision is made.

The system state at the beginning of period \( t \) can be characterized by every patient’s health state at the previous visit and the time since then, which is \((\tilde{s}_{i,1}, \tilde{s}_{i,2}, \ldots, \tilde{s}_{i,t})\), \((h_{1,t}, n_{1,t}), (h_{2,t}, n_{2,t}), \ldots, (h_{I,t}, n_{I,t})\), or equivalently the information vectors \((\mathbf{\pi}_{i,1}, \mathbf{\pi}_{i,2}, \ldots, \mathbf{\pi}_{i,T})\), i.e., the distribution of the true state of each patient (Rosenfield 1976, Lovejoy 1987).

### 4.3. Reward

Let \( b_k \) denote the quality of life (QoL) score associated with health state \( k \). In any period \( t \), the expected health reward for patient \( i \) with state \((h_{i,t}, n_{i,t})\) and corresponding information vector \( \mathbf{\pi}_{i,t} \) is given by

\[
\phi(\mathbf{\pi}_{i,t}) = \sum_{k=0}^{K-1} (\mathbf{\pi}_{i,t})_k b_k. \tag{2}
\]

Without loss of generality, we assume that \( b_0 \geq b_1 \geq \cdots \geq b_K \).

### 4.4. Capacity Allocation

We represent capacity allocation decisions by binary decision variables \( a_{i,t} \), where \( a_{i,t} = 1 \) if patient \( i \) is scheduled in period \( t \) and \( a_{i,t} = 0 \) otherwise. Denoting capacity in each period by \( C \) and assuming that all scheduled patients attend their appointments, the capacity constraint is given by

\[
\sum_{i=1}^{I} a_{i,t} \leq C, \quad \forall t \in \{1, 2, \ldots, T-1\}. \tag{3}
\]

Note that since the payoff is collected at the beginning of every period, for a \( T \) period problem, only decisions in the first \( T - 1 \) periods impact the objective function. Since treatment improves health states (i.e., \( \mathbf{c} \) is lower triangular), the capacity constraint (3) is binding at optimality. Thus, we consider only allocations that use all capacity. Let \( N \) denote the set containing all selections or subsets from index vector \( \{1, 2, \ldots, I\} \) with a cardinality of \( C \). An element \( N_i \in N \) represents a feasible capacity allocation rule in period \( t \).

We define the value function \( u^*_t(\mathbf{\pi}_{1,t}, \ldots, \mathbf{\pi}_{I,t}) \) as the total optimal discounted expected payoff from period \( t \) until the end of the horizon if the information vector for all patients at the beginning of period \( t \) is \((\mathbf{\pi}_{1,t}, \ldots, \mathbf{\pi}_{I,t})\). Puterman (1994) shows that for the discounted MDP, for any optimal policy, its induced value function must satisfy the following optimality equations:

\[
u^*_t(\mathbf{\pi}_{1,t}, \ldots, \mathbf{\pi}_{I,t}) = \sum_{i=1}^{I} \phi(\mathbf{\pi}_{i,t}) + \beta \max_{N_i \in N} \left\{ E\left[ u^*_{t+1}(\mathbf{\pi}_{1,t+1}, \ldots, \mathbf{\pi}_{I,t+1}) \mid N_i \right] \right\}, \tag{4a}
\]

\[
u^*_t(\mathbf{\pi}_{1,t}, \ldots, \mathbf{\pi}_{I,t}) = \sum_{i=1}^{I} \phi(\mathbf{\pi}_{i,t}), \tag{4b}
\]

where \( \beta \in (0, 1) \) is the discounting factor.
5. Analysis of the Optimal Policy

In this section, we establish the structural properties of the MDP model formulated in (4a)-(4b), which falls under the category of RMAB problems (Whittle 1988). The general RMAB problem is known to be PSPACE-complete (Papadimitriou and Tsitsiklis 1994), which makes it analytically intractable. Consequently, most of the effort in the literature has been devoted to developing heuristics, usually index policies (e.g., Whittle 1988, Bertsimas and Niño-Mora 2000). In contrast to this approach, we use the characteristics of our operational setting to derive the structure of the optimal policy and obtain managerial intuition for specific problems of interest.

In particular, we study a model with two health states in §5.1 and then extend the analysis to two specific problems with multiple health states in §5.2. In the first instance (§5.2.1), we assume that the treatment is perfect; i.e., a patient’s health state recovers to the best health state after treatment. In the second (§5.2.2), the treatment effect is more general but the number of available slots in each treatment. In the second (§5.2.2), the treatment effect is analytically intractable. Consequently, most of the effort in the literature has been devoted to developing heuristics, usually index policies (e.g., Whittle 1988, Bertsimas and Niño-Mora 2000). In contrast to this approach, we use the characteristics of our operational setting to derive the structure of the optimal policy and obtain managerial intuition for specific problems of interest.

In this section, we establish the structural properties of the optimal policy and obtain managerial intuition for specific problems of interest.

5.1. Two Health States

With two health states ($K = 2$), the information vector of patient $i$ at the beginning of period $t$ can be expressed as $\vec{\pi}_{i,t} = (\pi_{i,1}, \pi_{i,2})$, and $\pi_{i,1}$ is sufficient to characterize $\vec{\pi}_{i,t}$. To simplify notation, in this section, we define $\pi_{i,1} = (\pi_{i,1})_0$, i.e., the first element in the information vector, which we refer to as simply the information for patient $i$. Consequently, for the two health states model, the notation of the value function $u^*(\vec{\pi}_{i,1}, \pi_{i,2}, \ldots, \pi_{i,t})$ is simplified to $u^*(\pi_{i,1}, \pi_{i,2}, \ldots, \pi_{i,t})$.

Without loss of generality, let the disease progression matrix $\mathcal{D}$ and treatment matrix $\mathcal{E}$ be

$$\mathcal{D} = \begin{bmatrix} p_0 & 1 - p_0 \\ p_1 & 1 - p_1 \end{bmatrix}, \quad \mathcal{E} = \begin{bmatrix} 1 & 0 \\ q & 1 - q \end{bmatrix}.$$

In the following proposition, we establish the monotonicity and componentwise convexity properties of value function $u^*(\pi_{i,1}, \pi_{i,2}, \ldots, \pi_{i,t})$. The proofs of all theoretical results are provided in the appendix.

**Proposition 1.** If $p_0 > p_1$, then for all $i \in \{1, 2, \ldots, I\}$ and for all $t \in \{1, 2, \ldots, T\}$: (i) $u^*(\pi_{i,1}, \pi_{i,2}, \ldots, \pi_{i,t})$ is non-decreasing in $\pi_{i,t}$; and (ii) $u^*(\pi_{i,1}, \pi_{i,2}, \ldots, \pi_{i,t})$ is componentwise convex in $\pi_{i,t}$.

For our setting of asthma care, the two states can be interpreted as the patient’s symptoms being controlled or uncontrolled. Thus $p_0 > p_1$ implies that, without treatment, a patient is more likely to remain in the controlled state than to transition into the controlled state from an uncontrolled state.

Note that patients can be completely ranked by their information, i.e., the probability that a patient’s true health state is controlled. Intuitively one expects that under such complete ranking, it would be optimal to prioritize patients with worse information. This intuition is formally proven in Theorem 1 below.

**Theorem 1.** In any period $t \in \{1, 2, \ldots, T - 1\}$, without loss of generality, reindex the patients by their information such that $\pi_{i,1} \leq \pi_{j,1} \leq \cdots \leq \pi_{I,1}$. If $p_0 > p_1$, then it is optimal to schedule patients $1, 2, \ldots, C$.

5.2. Multiple Health States

Unlike the setting with two health states, there is no simple method to rank the information vectors of patients when $K > 2$. We begin by proposing the following ordering to rank two vectors:

**Definition 1.** Vector $\vec{y}_i$ of size $K$ dominates vector $\vec{y}_j$ if for any $l$ such that $0 \leq l < K - 1$, $\sum_{k=l+1}^{K-1}(\vec{y}_i)_k \leq \sum_{k=l+1}^{K-1}(\vec{y}_j)_k$, and for $l = K - 1$, $\sum_{k=l}^{K-1}(\vec{y}_i)_k = \sum_{k=l}^{K-1}(\vec{y}_j)_k$. We use $\vec{y}_i > \vec{y}_j$ to denote that $\vec{y}_i$ dominates $\vec{y}_j$.

This definition reduces to the usual first order stochastic dominance if $\vec{y}_i$ and $\vec{y}_j$ are probability distributions, which we denote as $\vec{y}_i \succ_{s.t.} \vec{y}_j$ (Shaked and Shanthikumar 2006).

In any period $t$, $\vec{\pi}_{i,t} >_{s.t.} \vec{\pi}_{j,t}$, implies that the probability of patient $i$ being in any health state $0 \leq k \leq K - 1$ or worse is higher than or equal to that of patient $j$. For ease of exposition, we refer to this relationship as “patient $i$’s health state is worse than that of patient $j$” or “patient $i$ is sicker and patient $j$ is healthier.”

In this section, we restrict both $\mathcal{D}$ and $\mathcal{E}$ to a set $\mathcal{W}$ defined by matrix $\mathcal{W} \in \mathcal{W}$ that satisfies Condition (C-1).

**Condition (C-1).** $\vec{w}_k \prec_{s.t.} \vec{w}_{k+1}$, $\forall k \in \{0, 1, \ldots, K - 2\}$, where $\vec{w}_k$ is the $k$th row of the matrix $\mathcal{W}$. When applied to a stochastic matrix (a matrix in which row vectors sum to 1), Condition (C-1) is often referred to as the stochastically increasing property (Derman 1963, Lovbjerg 1987).

Lemma 1 shows that matrix $\mathcal{W} \in \mathcal{W}$ that satisfies Condition (C-1) preserves stochastic ordering.

**Lemma 1.** Suppose matrix $\mathcal{W} \in \mathcal{W}$ satisfies Condition (C-1) and $\sum_{k=0}^{K-1}(\vec{w}_i)_k = 1$. Consider any two vectors $\vec{y}_i$ and $\vec{y}_j$ such that $\sum_{k=0}^{K-1}(\vec{y}_i)_k = \sum_{k=0}^{K-1}(\vec{y}_j)_k = 1$ and $(\vec{y}_i)_k > (\vec{y}_j)_k \geq 0$. Then $\vec{y}_i \succ_{s.t.} \vec{y}_j$ implies that $\vec{y}_i \succ_{s.t.} \vec{y}_j \bar{\mathcal{W}}$; i.e., $\mathcal{W}$ preserves stochastic order.

Note that Condition (C-1) is a direct extension of $p_0 > p_1$ and $q \leq 1$ in the two health state model. Specifically, for the model of multiple health states, consider patients $i$ and $j$ that are both not scheduled in period $t$. If $\mathcal{D}$ satisfies Condition (C-1), then the information vectors of these two patients have the same pairwise ranking in period $t + 1$ as that in period $t$. Similarly, consider patient $i$ and $j$ that are both scheduled in period $t$. If $\mathcal{E}$ satisfies Condition (C-1) and consequently $\mathcal{D} \mathcal{E}$ satisfies Condition (C-1), then the information vectors of these two patients will maintain their pairwise ranking.

Although Condition (C-1) preserves stochastic ordering of two patients’ information vectors, it is not sufficient to
fully characterize the optimal policy because the ranking of patients might not be complete (i.e., all patients cannot be reindexed as in Theorem 1). The reason is as follows. In any period $t$, in addition to changes because of disease progression and treatment, the information vector of a scheduled patient also changes because of diagnosis, which is modeled as a random event. If patient $i$ is treated in period $t$, then patient $i$'s true health state $h_{i,t}$ is realized at the beginning of period $t$ by diagnosis. Thus, the information vector of patient $i$ after diagnosis but before treatment is a deterministic vector $\bar{e}_{h_{i,t}}$. For $\bar{e}_{h_{i,t}}$ to be comparable with all other patients' information vectors, $\bar{e}_{h_{i,t}}$ must be comparable with all feasible information vectors for all $h_{i,t}$. We refer to this characteristic of the optimal policy as complete ranking.

Furthermore, even if all patients can be ranked, the optimal policy might not have the intuitive feature of scheduling the sicker patients first, given the definition of sicker patients explained above. We refer to the intuitive characteristic of the optimal policy as prioritizing sicker patients.

In the following sections, we restrict the discussion to two special cases where the optimal policy has these characteristics.

5.2.1. Perfect Treatment Process. Motivated by the assumption of perfect repair in the machine maintenance and repair literature (Block et al. 1993, Wang and Pham 1999, Pham and Wang 2000), we consider a perfect treatment matrix characterized by $\bar{e}_{\text{perfect}}$, in which $\bar{e}_{ii} = 1 \forall i$ and $\bar{e}_{ij} = 0 \forall i, \forall j \neq i$:

$$\bar{e}_{\text{perfect}} = \begin{bmatrix}
1 & 0 & \cdots & 0 \\
\vdots & \ddots & \ddots & \vdots \\
1 & 0 & \cdots & 0
\end{bmatrix} \quad (5)$$

Next, we establish a proposition that is parallel to Proposition 1, characterizing the monotonicity property of value function $u^*$($\bar{e}_{1,t}, \bar{e}_{2,t}, \ldots, \bar{e}_{i,t}$).

**Proposition 2.** If the disease progression matrix $\mathcal{P}$ satisfies Condition (C-1) and $\bar{e} = \bar{e}_{\text{perfect}}$, then $u^*($$\bar{e}_{1,t}, \bar{e}_{2,t}, \ldots, \bar{e}_{i,t}$) is componentwise decreasing in the sense that for all $\bar{e}_{i,t} > \bar{e}_{j,t}$, we have $u^*($$\bar{e}_{1,t}, \ldots, \bar{e}_{i,t}, \ldots, \bar{e}_{j,t}$) $\leq$ $u^*($$\bar{e}_{1,t}, \ldots, \bar{e}_{i,t}, \ldots, \bar{e}_{j,t}$).

The above proposition implies that for two systems with identical states of all patients except one ($\bar{e}_{i,t} > \bar{e}_{j,t}$), the system in which patient $i$ is worse ($\bar{e}_{h_{i,t}}$) has a lower aggregated QALY under the optimal policy. If patient $i$ is scheduled in period $t$, then regardless of his diagnosed health state $h_{i,t}$, his information vector at the beginning of period $t+1$ is $\bar{e}_{h_{i,t+1}} = \bar{e}_{h_{i,t}} \bar{P} = \bar{p}_i$. Hence $\mathcal{P}$ satisfying Condition (C-1) is enough to guarantee complete ranking. Theorem 2 shows that with perfect treatment effect $e_{\text{perfect}}$, this implies that the optimal policy prioritizes sicker patients.

**Theorem 2.** If the disease progression matrix $\mathcal{P}$ satisfies Condition (C-1) and $\bar{e} = \bar{e}_{\text{perfect}}$, then the information vectors of all patients can be completely ranked in any period $t$. Without loss of generality, reindex the patients such that $\bar{e}_{1,t} > \bar{e}_{2,t} > \cdots > \bar{e}_{i,t}$. Then the optimal policy schedules patient $1, 2, \ldots, C$.

5.2.2. General Treatment Process. In this section, we extend our analysis to accommodate a more general treatment matrix $\bar{e} \neq \bar{e}_{\text{perfect}}$. As noted earlier, the main analytical challenge lies in showing that all patients can be ranked by their information vectors and that prioritizing sicker patients is optimal. We illustrate this challenge in the example below.

**Example 1.** Consider the following disease progression and treatment matrices with three health states:

$$\mathcal{P}_1 = \begin{bmatrix}
0.9 & 0.1 & 0 \\
0.5 & 0.3 & 0.2 \\
0.2 & 0.2 & 0.6
\end{bmatrix}, \quad \bar{e}_1 = \begin{bmatrix}
1 & 0 & 0 \\
0.7 & 0.3 & 0 \\
0.5 & 0.2 & 0.3
\end{bmatrix}.$$  

Note that both $\mathcal{P}_1$ and $\bar{e}_1$ satisfy Condition (C-1). Suppose that patient $i$ was last seen at period $t$ and was diagnosed in health state 1 and patient $j$ was last seen in period $t-1$ and was diagnosed in health state 1. Further, suppose that patient $i$ is scheduled and patient $j$ is not scheduled in period $t$. Patient $i$'s information vector in period $t+1$ is $\bar{e}_{i,t+1} = \bar{e}_1 \bar{P} = \bar{e}_{\text{perfect}} \bar{P} = \bar{p}_i$, and patient $j$'s information vector in period $t+1$ is $\bar{e}_{j,t+1} = \bar{e}_1 \bar{P} = \bar{e}_{\text{perfect}} \bar{P} = \bar{p}_j$. We show that $\bar{e}_{i,t+1} > \bar{e}_{j,t+1}$.

Note that patient $j$ has higher probability of transitioning to both health states 0 and 2 than does patient $i$. Hence, $\bar{e}_{i,t+1}$ and $\bar{e}_{j,t+1}$ cannot be ordered according to first order stochastic dominance.

To avoid cases of incomplete ordering, we impose an additional condition on matrix $\bar{e}$ (for a given matrix $\mathcal{P}$) such that the information vectors of patients $i$ and $j$ are still ordered after the realization of the random event corresponding to the diagnosis of the true health state for one of these two patients.

**Condition (C-2).** $\bar{e}_{K-1,t} \sim_{\mathcal{P}} \bar{p}_0$, where $\bar{e}_{K-1,t}$ is the $K$th row of $\bar{e}$ and $\bar{p}_0$ is the first row of $\mathcal{P}$.

Combined with $\mathcal{P}$ and $\bar{e}$ satisfying Condition (C-1), Condition (C-2) implies that the worst possible information vector of patient $j$ in period $t+1$ if she was scheduled in period $t$ (i.e., $\bar{e}_{K-1,t} \mathcal{P}$) is better than the best possible information vector of patient $i$ in period $t+1$ if she was not scheduled in period $t$ (i.e., $\bar{e}_i \mathcal{P} = \bar{p}_i \mathcal{P}$). This allows all feasible information vectors to be ranked, regardless of the scheduling decisions. Condition (C-2) is satisfied for the perfect treatment matrix $\bar{e}_{\text{perfect}}$ for any $\mathcal{P}$ that satisfies Condition (C-1) and thus represents a more general setting than perfect repair. Proposition 3 formally states that $\mathcal{P}$ and $\bar{e}$ satisfying Condition (C-1) and Condition (C-2) are sufficient to ensure complete ranking.
Proposition 3. Suppose matrices $\mathcal{P}$ and $\mathcal{C}$ both satisfy Condition (C-1) and $\mathcal{C}$ satisfies Condition (C-2); then we have complete ranking of all patients’ information vectors. That is, in any period $t$, any two information vectors $\vec{\pi}_{i,t}$ and $\vec{\pi}_{j,t}$ are comparable using first order stochastic dominance; i.e., either $\vec{\pi}_{i,t} \succeq_{st} \vec{\pi}_{j,t}$ or $\vec{\pi}_{j,t} \succeq_{st} \vec{\pi}_{i,t}$.

Although it is intuitively appealing to schedule a sicker patient before a healthier patient, properties of $\mathcal{P}$ and $\mathcal{C}$ with respect to Conditions (C-1) and (C-2) above cannot guarantee that such a schedule policy is optimal. Consider the next example.

**Example 2.** Consider the following matrices $\mathcal{P}_2$ and $\mathcal{C}_2$:

$$
\mathcal{P}_2 = \begin{bmatrix}
0.5 & 0.4 & 0.1 \\
0.4 & 0.5 & 0.1 \\
0.4 & 0.4 & 0.2 \\
\end{bmatrix}, \quad \mathcal{C}_2 = \begin{bmatrix}
1 & 0 & 0 \\
0.9 & 1 & 0 \\
0.5 & 0.4 & 0.1 \\
\end{bmatrix}.
$$

Note that $\mathcal{P}_2$ and $\mathcal{C}_2$ both satisfy Condition (C-1) and $\mathcal{C}_2$ satisfies Condition (C-2) given $\mathcal{P}_2$. By Proposition 3, at any period $t$, all patients’ information vectors can be completely ranked. Consider a single period problem where the QoL scores are $(b_h, b_j, b_k) = (1, 0.2, 0.1)$. Suppose patient $i$’s initial health state is $h_{i,1} = 1$ and patient $j$’s initial health state is $h_{j,1} = 2$. The reward for scheduling patient $j$ is 1.38 and that for scheduling patient $i$ is 1.36. Thus, it is not optimal to schedule patient $i$ who is in a worse health state.

Consider a set $\mathcal{W}$ such that any matrix $\mathcal{W} \in \mathcal{W}$ satisfies Condition (C-3).

**Condition (C-3).** $\vec{w}_k \succeq_{st} \vec{w}_{k+1}$, $\forall k \in \{0, 1, \ldots, K-2\}$, where $\vec{w}_k$ is the $k$th row of $\mathcal{W}$. In the appendix we show that if matrix $\mathcal{W}$ satisfies Condition (C-3), then for any $\vec{\theta} = (\vec{\theta}_0, \ldots, \vec{\theta}_{K-1})$ such that $\vec{\theta}_0 \succeq_{st} \cdots \succeq_{st} \vec{\theta}_{K-1}$, the vector $\mathcal{W}^{\vec{\theta}}$ is increasing in its elements (Lemma 8). Note that (C-3) is satisfied for $\mathcal{C}_2$ for any $\mathcal{P}$ that satisfies (C-1).

Next, we show that prioritization of sicker patients is optimal for matrices $\mathcal{C} \mathcal{P} - \mathcal{P}$, $\mathcal{C} \mathcal{P} - \mathcal{C} \mathcal{P} \in \mathcal{W}$. Consider patient $i$ with information vector $\vec{\pi}_{i,t}$, and patient $j$ with information vector $\vec{\pi}_{j,t}$. Patient $i$’s expected QALY in the next period is $\vec{\pi}_{i,t,\mathcal{C} \mathcal{P}}^{\vec{\theta}^T} \mathcal{P} \vec{\theta}^T$ if he is seen and $\vec{\pi}_{i,t,\mathcal{C} \mathcal{P}}^{\vec{\theta}^T} \mathcal{P} \vec{\theta}^T$ if he is not seen, where $\vec{\theta} = (b_0, \ldots, b_{K-1})$ is the vector of QoL scores. Thus, $\vec{\pi}_{i,t,\mathcal{C} \mathcal{P}}^{\vec{\theta}^T} \mathcal{P} \vec{\theta}^T$ represents the marginal benefit of seeing patient $i$. If matrix $\mathcal{C} \mathcal{P} - \mathcal{P}$ satisfies Condition (C-3), then $\vec{\pi}_{i,t,\mathcal{C} \mathcal{P}}^{\vec{\theta}^T} \mathcal{P} \vec{\theta}^T \succeq_{st} \vec{\pi}_{j,t,\mathcal{C} \mathcal{P}}^{\vec{\theta}^T} \mathcal{P} \vec{\theta}^T$, which implies that the marginal benefit of next period’s expected QALY for patient $i$ is smaller if patient $i$ is healthier.

Similarly, $\vec{\pi}_{i,t,\mathcal{C} \mathcal{P} - \mathcal{C} \mathcal{P}}^{\vec{\theta}^T} \mathcal{P} \vec{\theta}^T$ represents the marginal benefit of seeing patient $i$ if period one period earlier. Given that $(\mathcal{C} \mathcal{P} - \mathcal{C} \mathcal{P})$ satisfies Condition (C-3), we have $\vec{\pi}_{i,t,\mathcal{C} \mathcal{P} - \mathcal{C} \mathcal{P}}^{\vec{\theta}^T} \mathcal{P} \vec{\theta}^T \succeq_{st} \vec{\pi}_{j,t,\mathcal{C} \mathcal{P} - \mathcal{C} \mathcal{P}}^{\vec{\theta}^T} \mathcal{P} \vec{\theta}^T$.

In other words, the marginal benefit of seeing patient $j$ one period earlier is greater than that of patient $i$.

**Theorem 3.** Consider a system with $I$ patients, $1$ slot, and $K$ health states in which the disease progression matrix $\mathcal{P}$ and the treatment matrix $\mathcal{C}$ satisfy Conditions (C-1) and $\mathcal{C}$ satisfies Condition (C-2), matrices $\mathcal{C} \mathcal{P} - \mathcal{P}$ and $\mathcal{C} \mathcal{P} - \mathcal{C} \mathcal{P}$ both satisfy Condition (C-3); then if the information vectors of all patients in period $t$ are ranked such that $\vec{\pi}_{1,t} \succeq_{st} \cdots \succeq_{st} \vec{\pi}_{I,t}$, the optimal policy schedules patient $1$ in period $t$.

Observe that as compared to Theorem 2, the result in Theorem 3 is limited to the case when there is only one slot available. However, since $\mathcal{C}_1$ satisfies (C-3) for any $\mathcal{P}$ that satisfies (C-2), Theorem 3 represents a generalization of Theorem 2 with respect to the effect of treatment.

**6. Myopic Heuristic**

Theorems 1–3 characterize a method to schedule all patients optimally in special settings. However, some problem settings may not fall into these categories. In this section, we develop a heuristic for such situations.

We firstly consider a myopic policy that schedules patients to maximize the community’s aggregated QALYs only in the immediate next period after the scheduling decision. This is motivated by the strong performance of such policies in multiarmed bandit problems; see Ryzhov et al. (2010, 2012) and Ryzhov and Powell (2009). Formally, at the beginning of period $t$, the myopic policy determines a selection $N_t \in N$ of patients to be seen in period $t$, which is the optimal solution to the following static optimization problem:

$$
\max_{N_t \subseteq N} \left[ \sum_{i=1}^{I} \phi(\vec{\pi}_{i,t+1}) \right] \vec{\pi}_{i,t}, \vec{\pi}_{2,t}, \ldots, \vec{\pi}_{I,t}.
$$

The following lemma gives the structure of the myopic policy.

**Lemma 2.** In any period $t \in \{1, \ldots, T-1\}$, define an index for each patient,

$$
\Psi_{i,t}(\vec{\pi}_{i,t}) = \sum_{k=0}^{K-1} \left[ (\vec{\pi}_{i,t})_{k} \phi(\bar{e}^{\kappa} \mathcal{P} - \phi(\vec{\pi}_{i,t})) \right].
$$

Without loss of generality, reorder the patients according to their indices; i.e., $\Psi_{1,t} \geq \Psi_{2,t} \geq \cdots \geq \Psi_{C,t} \geq \Psi_{C+1,t} \cdots \geq \Psi_{I,t}$. The myopic policy schedules patients $1, 2, \ldots, C$.

Lemma 2 shows that the myopic policy has the form of an index policy: patients are ranked at the beginning of period $t$ based on indices given by $\Psi_{i,t}$ as defined in (6), which corresponds to the marginal benefit of scheduling patient $i$ in period $t+1$. For all the three settings discussed in §5, if the information vector of patient $i$ stochastically dominates the information vector of patient $j$ at the beginning of period $t$, the marginal benefit in period $t$ is larger if patient $i$ is scheduled instead of patient $j$. Therefore, the myopic policy and the optimal policy coincide as long as all patients’ information vectors can be ranked, as shown in Theorem 4.
The myopic policy schedules the same patients as the optimal policy in the following cases:

- \( K = 2 \) health states, 1 patients, and C appointment slots;
- \( K > 2 \) health states, 1 patients, C appointment slots, \( P \) satisfies Condition (C-1) and perfect treatment, i.e., \( \Psi = \Psi_{\text{perfect}} \);
- \( K > 2 \) health states, 1 patients, 1 appointment slot and \( \Psi \) and \( \Phi \) satisfy Condition (C-2), \( \Phi \) satisfies Condition (C-2), and \( \Phi = P \) and \( \Phi = P \) both satisfy Condition (C-3).

The computationally simpler myopic policy produces the optimal schedule for each one of the above settings. More importantly, when the information vectors of all patients are not fully ranked, i.e., when we cannot rank all patients’ using the first order stochastic dominance of their information vectors, the myopic policy still generates a complete ordering of all patients using indices \( \Psi_{i,t+1} \). Hence myopic policy can be used to generate a feasible schedule that can be used as a heuristic scheduling policy. We define the myopic heuristic formally below:

**Myopic heuristic for scheduling patients in period** \( t \in [1, 2, \ldots, T - 1] \).

1. Calculate the information vector \( \tilde{\pi}_{i,t} \), based on \( h_{i,t} \) and \( n_{i,t} \), for all patients \( i \in [1, 2, \ldots, I] \).
2. Calculate the marginal benefit \( \Psi_{i,t}(\tilde{\pi}_{i,t}) \) for all patients \( i \in [1, 2, \ldots, I] \).
3. Schedule all patients by \( \Psi_{i,t} \) in descending order.
4. Schedule the first 4 patients from the above ranking.

We study the performance of the myopic heuristic in §8.2 for more general settings, when Conditions (C-1), (C-2), and (C-3) do not hold.

### 7. Model Calibration

In this section, we calibrate our model using operational and clinical data from Mobile C.A.R.E. Foundation regarding its school-based asthma care for children. Founded in 1998, MCF has partnered with more than 100 schools and Head Start programs, assisted in the screening of about 50,000 children for asthma, and treated nearly 5,000 patients. We describe the data in §7.1 and provide a qualitative and quantitative characterization of the current capacity allocation process at MCF in §7.2. In §7.3, we estimate the disease progression and treatment matrices of our model. These form the basic building blocks for calculating the potential benefit of the integrated allocation approach over the current practice.

#### 7.1. Data

We utilize a comprehensive data set consisting of data that are routinely collected during patient visits to the vans using an electronic medical records system called AsmaTrax® (see Jones et al. 2007). It comprises 29,745 observations (appointments/visits) for 5,041 individual patients, over 10 years between 1999 and 2009. During each patient visit, the medical staff records a variety of clinical indicators self-reported by patients and caregivers, including the occurrence of day and night time symptoms, FEV1% (Forced Expiratory Volume in 1 second) measured using spirometry, number of school days missed, and number of hospital visits. These data are used to characterize patient health along two dimensions—severity and control. Severity captures the inherent intensity of the disease process in a patient, which typically remains unchanged across visits. MCF, consistent with the broader clinical literature, maintains four categories: mild intermittent, mild persistent, moderate persistent, and severe persistent. Control indicates how well asthma-related symptoms are currently controlled in a patient. MCF maintains four categories: controlled, improved, unchanged, and worsened, where the last three fall under the umbrella of uncontrolled.

#### 7.2. Characterizing Current Policy at MCF

We present a qualitative overview of the current capacity allocation policy at MCF in §7.2.1, followed by its quantitative characterization in §7.2.2 using MCF data.

##### 7.2.1. Qualitative Overview of the Current Practice

The current MCF scheduling process is shown in Figure 1 and described in greater detail in Deo et al. (2009). Notably, scheduling is performed in two steps. First, physicians recommend a due date for the next visit that is primarily driven by a patient’s control status. According to the MCF medical team, the recommended interval between current and next visit is three months for controlled category of patients and one month for the uncontrolled categories.

Two to four weeks before a school visit, based on the physician assignments and available van capacity (typically 16 slots), schedulers at the MCF office develop a feasible allocation of capacity among the school population for that visit. In this scheduling process, priority is given in the following order: (1) patients who request appointments, (2) patients who could not be seen previously due to capacity limits, (3) patients who have missed prior appointments, (4) all other patients who are due for an appointment, and (5) new patients.

This two-step process with disjoint operational and clinical considerations is driven by a lack of a systematic framework to integrate capacity constraints directly in the recommended intervals between visits in addition to an inadequate IT system. MCF management is interested in understanding if this disjoint decision making process leads to stable patients being scheduled more frequently than necessary, thereby resulting in reduced program effectiveness.
7.2.2. Quantitative Analysis of the Current Practice. Based on our qualitative understanding of the current practice, we investigate how the duration between visits quantitatively depends on two key dimensions of patient severity and control type. We conduct an ordinary least squares (OLS) regression of the square root transformation of our dependent variable (duration between visits) against these independent variables and control for whether the previous appointment was kept or not and for the year of the visit. For this analysis, we consider data from the years 2006 to 2009 consisting of 15,859 appointments of 2,840 patients since records in this period distinguish between scheduled appointments and actual visits. Our results confirm two main aspects of the qualitative characterization. First, a worse control category is consistently associated with smaller time between appointments. This is consistent with physicians’ medically recommended intervals. Second, the time to the next appointment is smaller if the previous appointment was missed than if it was kept. This is consistent with the MCF schedulers’ prioritization. For more details, see Table 7 in Appendix A.

7.2.3. A Baseline Characterization of the Current Practice. Although the statistical significance for the key independent variables in our OLS regression confirms our qualitative findings, the low value of $R^2$ precludes direct application of the regression results to calculate the duration between visits. Therefore, we construct the following baseline version of MCF’s current practice. At each visit, the patient is assigned a follow-up date consistent with the medically recommended intervals. These follow-up dates are modified to ensure feasibility and to capture the current prioritization method at MCF as follows. If the number of patients due for a visit exceeds capacity on a particular day, priority is given to patients who were due back in prior periods but not scheduled due to capacity constraints followed by patients who are due back in the current period. Within each group, patients are selected randomly if the group size exceeds available capacity. If the capacity exceeds the number of patients due for a visit, patients who are due back in future periods are scheduled to use all capacity. In what follows, we refer to this as the fixed duration policy.

7.3. Characterizing Disease Progression

We investigate the drivers of disease progression to obtain an appropriate definition of health state (§7.3.1) and estimate transition probabilities over these health states (§7.3.2).

7.3.1. Drivers of State Transition. We estimate a multinomial regression model to estimate the likelihood of patients moving from one control type to another depending on the underlying severity, time since last appointment, calendar year, season of the year, and whether the school is in session or not. To be consistent with §7.2, we focus on patient visits data from 2006 onward that consist of 7,431 visits and 1,473 patients. The results are shown in Tables 8 to 11 in Appendix A. We find that time since last appointment and severity type have a significant impact on the likelihood of transition among control types. This indicates that the transition probability matrices across control types should be estimated separately for different severity levels. We describe our estimation procedure next.

7.3.2. Estimation of the Transition Probability Matrix. With common observation intervals for all patients, one can estimate the transition probability $P_{ij}$ simply by the ratio of number of patients moving from state $i$ to state $j$ to the total number of patients moving out of state $i$ during that observation interval. However, observation intervals in our data set vary across patients. This can be interpreted as a missing data situation; i.e., we do not observe the state of the patients in the intervening periods. We overcome this obstacle by using the Expectation-Maximization (EM) algorithm (Dempster et al. 1977) that iteratively estimates the missing data and then maximizes the likelihood of observing these data. Specifically, we adopt the approach in Craig and Sendi (2002) to obtain disease progression matrices of discrete time Markov chains as follows.

We fix one month as the cycle length, which is the underlying time scale over which state transitions occur. We round the actual observation intervals to integer multiples, $n$, of the cycle length. For a patient transitioning from state $i$ to state $j$ over $n$ cycles, transitions occurring during the intervening cycles are unobserved and are treated as missing data. The expectation part of the EM algorithm
### Table 1. Transition probability matrix $P$ for natural disease progression.

<table>
<thead>
<tr>
<th></th>
<th>Mild intermittent</th>
<th>Mild persistent</th>
<th>Moderate persistent</th>
<th>Severe persistent</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>C</td>
<td>I</td>
<td>U</td>
<td>W</td>
</tr>
<tr>
<td>C</td>
<td>0.97</td>
<td>0.01</td>
<td>0.01</td>
<td>0.02</td>
</tr>
<tr>
<td>I</td>
<td>0.00</td>
<td>0.85</td>
<td>0.08</td>
<td>0.07</td>
</tr>
<tr>
<td>U</td>
<td>0.00</td>
<td>0.00</td>
<td>1.00</td>
<td>0.00</td>
</tr>
<tr>
<td>W</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>1.00</td>
</tr>
</tbody>
</table>


Imputes the states at these unobserved cycles and tallies the corresponding expected number of transitions. The maximization part of the EM algorithm uses the method of common observation intervals described above to estimate the transition probabilities. These steps are repeated iteratively until the estimates converge. We use different initial values to test the robustness of the algorithm’s results.

We use the EM algorithm separately for each severity level to generate four matrices of transition probabilities $P$ and four treatment matrices $Q$. We restrict $P$ to be upper triangular and $Q$ to be a lower triangular matrix to enforce that (i) natural disease progression cannot improve the health state by itself and (ii) treatment, in isolation, cannot worsen the patients’ health status by itself. The estimates are shown in Tables 1 and 2. To confirm the Markovian property, we repeat the same analysis with a cycle length of two months and check that the two-month transition matrix is similar to the square of the one-month transition matrix.

The matrices in Tables 1 and 2 exhibit intuitive behavior (e.g., the probability of moving to the controlled state with treatment decreases with the initial health state); however, the matrices do not fully preserve stochastic ordering as defined by Conditions (C-1) and (C-2). The pairwise ordering of rows comprising Conditions (C-1) and (C-2) is satisfied for most pairs of rows, but not all. This precludes the implementation of the optimal policy described in Theorem 2, and hence we use the myopic heuristic as a mechanism to improve capacity allocation for MCF.

### Table 2. Transition probability matrix $Q$ for treatment.

<table>
<thead>
<tr>
<th></th>
<th>Mild intermittent</th>
<th>Mild persistent</th>
<th>Moderate persistent</th>
<th>Severe persistent</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>C</td>
<td>I</td>
<td>U</td>
<td>W</td>
</tr>
<tr>
<td>C</td>
<td>1.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>I</td>
<td>0.72</td>
<td>0.28</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>U</td>
<td>0.70</td>
<td>0.21</td>
<td>0.09</td>
<td>0.00</td>
</tr>
<tr>
<td>W</td>
<td>0.31</td>
<td>0.58</td>
<td>0.07</td>
<td>0.04</td>
</tr>
</tbody>
</table>

8. Computational Study

In this section, we report the results of our extensive computational study with three main objectives. First, we compare the performance of the myopic heuristic with that of the optimal policy (§8.2) and Whittle’s index (§8.3). Second, we quantify the extent of improvement obtained by using the myopic heuristic over the fixed duration policy (§§8.4–8.6). Third, we characterize how the myopic heuristic improves community access to care compared to the fixed duration policy through more effective capacity allocation (§§8.5 and 8.6). We start by describing the design of experiments and the associated parameter values in §8.1.

8.1. Design of Experiments

For our computational experiments, we consider a finite horizon of two years comprising 24 one-month periods. A month is of the same order of magnitude as the typical duration between two successive van visits to a school. Moreover, the recommended interval between patient visits is typically an integer multiple of a month. We assume the discounting factor to be 1.

Our experiments consist of two parts: (i) comparison of the myopic heuristic with the optimal policy and the Whittle’s index using dynamic programming and (ii) comparison of the myopic heuristic and the fixed duration policy using simulation. We first describe the choice of parameter values that are common to both parts followed by those that are different. All parameters are listed in Table 3.

Quality of Life scores. We utilize the clinical literature to derive estimates for QoL scores. Specifically, we use QoL estimates from Briggs et al. (2006). They use validated questionnaires to obtain survey responses from patients or caregivers and then convert the questionnaire responses into utility scores for four different control types. Since their control types are slightly different from MCF’s classification, we construct three alternative sets of QoL scores by fixing the utility levels for the healthiest state (state 0) and worst state (state 3) and interpolating between them. We label the sets as convex, linear, and concave depending on how the QoL scores change over the health states.

Transition probabilities. For the disease progression \( \mathcal{P} \) and treatment \( \mathcal{E} \) matrices, we use the estimated results from §7.3 for the four different severity levels.

History of past visits. In addition to the health state at the previous visit, another dimension of our state space is the time since last visit \( n_{i,t} \in \{1, 2, \ldots, N\} \). For a finite time horizon of \( T = 24 \) periods, \( N = T - 1 = 23 \). However, because of memory constraints, we limit the history to be within \( N = 4 \) for our dynamic programming experiments. We do not impose any such restriction for simulation and let \( N = T - 1 \).

Capacity and cohort size. Because of the curse of dimensionality in dynamic programming, we consider a small cohort of 5 patients and choose capacity levels of 1, 2, and 3 to construct scenarios of varying tightness in capacity. For simulation, we consider a more realistic scenario with 50 patients and capacity levels of 5, 10 and 15 to reflect different patient-capacity ratios.

Initial state. The initial state consists of two main components—control state at the previous visit and time since the last visit. We construct three initial profiles of control types (best, medium, worst) depending on the number of patients in each state. For the dynamic program, we assume that all patients were seen any-where between 1 and 10 periods ago according to a general distribution.

Severity levels. For the dynamic program, we study four cohorts, each belonging to a different severity level. For the simulation, we consider one unified cohort with a severity profile that roughly matches the profile of all visits at MCF.

8.2. Suboptimality of the Myopic Heuristic

In this section, we report results of the dynamic program. We follow a three-step procedure to calculate the optimality gap for the myopic heuristic. We calculate the gross objective value (QALYs) corresponding to the optimal policy (QALY$_{\text{opt}}$), myopic heuristic (QALY$_{\text{myo}}$), and a baseline

<table>
<thead>
<tr>
<th>Table 3. Parameter values.</th>
<th>Parameter</th>
<th>Cases</th>
<th>Dynamic program (§8.2)</th>
<th>Simulation (§§8.4–8.6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of patients ((I))</td>
<td>5</td>
<td>50</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Capacity ((C))</td>
<td>Low</td>
<td>1</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Medium</td>
<td>2</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td></td>
<td>High</td>
<td>3</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td>Length of horizon ((T))</td>
<td>24</td>
<td>24</td>
<td></td>
<td></td>
</tr>
<tr>
<td>History ((N))</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Initial distribution of control types</td>
<td>Best</td>
<td>{1, 1, 1, 1}</td>
<td>{10, 10, 10, 10}</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Medium</td>
<td>{0, 1, 1, 3}</td>
<td>{0, 10, 15, 25}</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Worst</td>
<td>{0, 0, 4, 1}</td>
<td>{0, 0, 40, 10}</td>
<td></td>
</tr>
<tr>
<td>Duration since last visit</td>
<td>4 for all patients</td>
<td></td>
<td>Distribution over {1, 10}</td>
<td></td>
</tr>
<tr>
<td>Severity levels</td>
<td>Mild intermittent</td>
<td>{5, 0, 0, 0}</td>
<td>{9, 23, 15, 3}</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Mild persistent</td>
<td>{0, 5, 0, 0}</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Moderate persistent</td>
<td>{0, 0, 5, 0}</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Severe persistent</td>
<td>{0, 0, 0, 5}</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Disease progression matrix ((\mathcal{P}))</td>
<td>Estimate from MCF data</td>
<td>(Table 1)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Treatment matrix ((\mathcal{E}))</td>
<td>Estimate from MCF data</td>
<td>(Table 2)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Quality of life scores ((\tilde{b}))</td>
<td>Concave</td>
<td>{0.95, 0.90, 0.84, 0.73}</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Linear</td>
<td>{0.95, 0.87, 0.80, 0.73}</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Convex</td>
<td>{0.95, 0.82, 0.76, 0.73}</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note. Columns 3 and 4 reflect that some parameter values might be different in the dynamic programming and simulation computations.
of not scheduling any patients ($QALY_{\text{base}}$). The net benefit of each policy ($QALY$s gained) is given by $QALY_{\text{opt}} - QALY_{\text{base}}$ and $QALY_{\text{myo}} - QALY_{\text{base}}$. We calculate the optimality gap as the percentage loss in net benefit by using myopic heuristic compared to the optimal policy, given by

$$
\delta_1 = \frac{(QALY_{\text{opt}} - QALY_{\text{base}}) - (QALY_{\text{myo}} - QALY_{\text{base}})}{QALY_{\text{opt}} - QALY_{\text{base}}}
$$

Table 4 shows various statistics of this optimality gap for 108 instances (36 instances for each capacity level) of the problem corresponding to different combinations of initial states, QoL scores, and severity levels. This leads to several important observations.

**Observation 1 (Overall Performance).** Over all instances, the average optimality gap is 0.40%. Moreover, the maximum optimality gap is less than 3% and in more than 80% of the instances (88 out of 108), the optimality gap is less than 1%.

**Observation 2 (Impact of Capacity).** The performance of the myopic heuristic improves as the capacity becomes less tight, as can be seen from the average and maximum gaps.

### 8.3. Comparison of the Myopic Heuristic with Whittle’s Index

As discussed in §3, our capacity allocation problem is an instance of the RMAB problem, which has been widely solved in the literature using Whittle’s index. Hence, we benchmark the performance of the myopic heuristic with that of Whittle’s index for the instances in Table 4. The average gap for Whittle’s index is 0.22% across all instances, with average gaps of 0.24%, 0.30%, and 0.11% for capacity levels 1, 2, and 3, respectively. Thus, the performances of the myopic heuristic and Whittle’s index are quite comparable, with the latter being slightly better for smaller capacities and the former being slightly better for larger capacities. We comment on the comparative performance of the two heuristics for larger capacity values in §8.4.

Although yielding similar performance, Whittle’s index is significantly more difficult to compute than is the myopic heuristic. First, Whittle’s index involves repeated solution of a (single patient) dynamic program through a value iteration algorithm, which requires an advanced computing environment such as MATLAB® or C++. On the other hand, the myopic heuristic can be easily implemented in Microsoft Excel®. Second, the time required to generate Whittle’s index for all states is significantly larger than that for the myopic index, especially for problem instances with more than 10 health states, and increases rapidly with the number of health states thereafter. As an example, for an instance with 16 health states and a time horizon of 24 months (periods), it takes 32 hours to generate Whittle’s index whereas it takes less than 1 second to generate the myopic index. These computational issues are likely to create barriers to implementation of Whittle’s index in a nonprofit organization such as MCF that does not have the requisite monetary and human resources to manage this level of complexity. In addition to the computational simplicity, the easy interpretation of the myopic index—the benefit foregone for a particular patient by not seeing her in the next period—can further improve the chances of a successful implementation. Hence, we propose the myopic heuristic as a new approach to capacity allocation at MCF and evaluate its performance relative to a fixed duration policy in the subsequent sections.

### 8.4. Improvement Over the Fixed Duration Policy Using the Myopic Heuristic

We report the magnitude of potential improvement from implementing the myopic heuristic over the fixed duration policy for 27 combinations of capacity, QoL scores, and initial states in Table 5. Similar to the optimality gap in §8.2, we calculate this improvement as

$$
\delta_2 = \frac{(QALY_{\text{myo}} - QALY_{\text{base}}) - (QALY_{\text{fix}} - QALY_{\text{base}})}{QALY_{\text{fix}} - QALY_{\text{base}}}
$$

**Observation 3 (Overall Improvement).** MCF can obtain significant benefit by implementing the myopic heuristic, ranging between 3% and 16% for the parameter values considered.

Notably, the potential improvement in health outcomes is driven solely by more effective operational decisions. Higher effectiveness is achieved in our approach by utilizing both clinical (disease progression) and operational (capacity constraint) data to make operational decisions in an integrated manner. This is in contrast to the decoupled approach followed currently at MCF, described in §7.2.
Table 5. Performance improvement over the fixed duration policy obtained by using the myopic heuristic.

<table>
<thead>
<tr>
<th>QoL</th>
<th>Initial state</th>
<th>Capacity (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>5</td>
</tr>
<tr>
<td>Convex</td>
<td>Worst</td>
<td>11.33</td>
</tr>
<tr>
<td></td>
<td>Medium</td>
<td>9.86</td>
</tr>
<tr>
<td></td>
<td>Best</td>
<td>13.89</td>
</tr>
<tr>
<td>Linear</td>
<td>Worst</td>
<td>12.74</td>
</tr>
<tr>
<td></td>
<td>Medium</td>
<td>12.34</td>
</tr>
<tr>
<td></td>
<td>Best</td>
<td>14.49</td>
</tr>
<tr>
<td>Concave</td>
<td>Worst</td>
<td>14.28</td>
</tr>
<tr>
<td></td>
<td>Medium</td>
<td>14.22</td>
</tr>
<tr>
<td></td>
<td>Best</td>
<td>15.47</td>
</tr>
</tbody>
</table>

MCF might incur some nonmonetary adjustment costs in adopting the proposed policy and some upfront investment to modify its electronic medical records system. However, this change does not require additional recurring cost. Hence, the incremental cost effectiveness of our recommendation, the standard criterion used to evaluate healthcare interventions, is quite attractive. Moreover, it does not require changes to the core clinical practice such as therapy, diagnosis, etc., and hence would receive less resistance from physicians.

Appendix B introduces other easy-to-implement and intuitive methods to allocate capacity (e.g., scheduling patients by a prioritized ranking based on time since last visit or health state at last visit). The computational tests show that these policies do not yield the benefits of the myopic heuristic. In fact, in some settings, the policies perform worse than the fixed duration policy. Appendix B also includes performance improvement from using Whittle’s index. We find that despite the computational complexity of calculating the Whittle’s index, its performance is only marginally better than the myopic heuristic (on average 0.7 percentage points higher). This provides additional evidence for the attractiveness of using the myopic heuristic for our problem setting.

Impact of capacity. Owing to economic downturn and consequent reduction in funding, MCF has reduced the number of vans from 3 to 2 in 2009, thus significantly constraining its capacity relative to the patient population. Motivated by this event, we investigate the improvement in performance obtained from the myopic heuristic for different capacity levels.

Observation 4 (Impact of Capacity). Potential benefit from implementing the myopic heuristic is higher for tighter capacity and decreases as capacity increases.

This observation implies that when capacity tightens, MCF can incur a significant loss of welfare with the fixed duration policy (recommended medical duration between visits). Notably, the gap between the myopic heuristic and the fixed duration policy is much more sensitive to changes in capacity than is the gap between the optimal policy and myopic heuristic. This is because the myopic heuristic inherently adjusts the frequency of visits to varying capacity levels and prioritizes patients by control status when capacity is limited, whereas such an adjustment is post facto in the fixed duration policy only to ensure feasibility. We explore this issue further in §8.6.

Sensitivity to QoL scores. The three sets of QoL scores implicitly reflect different definitions of control types. For instance, the convex QoL set reflects that the best control type enjoys a much higher quality of life than all other control types, which are more similar to each other. On the other extreme, the concave QoL set reflects that the worst control type enjoys a much lower quality of life than all other control types.

Observation 5 (Impact of QoL Scores). Performance improvement for the concave QoL set is higher (than for the linear or convex set) for smaller capacity cases but lower for larger capacity cases.

Our implementation of MCF’s current policy recommends equal duration between visits for the three worse states and a much longer duration for the best state. Thus, we expect that the potential benefit should be higher under concave QoL set than convex or linear QoL set. The above observation confirms this intuition for small capacity cases.

Sensitivity to initial state. Table 5 shows that the magnitude of potential improvement is similar across different initial states (control types) of the patient population. The general trend indicates that, per intuition, the magnitude of improvement is mostly higher if more patients start in a worse health state. This is summarized in the following observation.

Observation 6 (Sensitivity of Initial State). Performance improvement is robust to changes in the initial state; i.e., distribution of patients’ health states at previous visits.

8.5. Improved Capacity Allocation

To understand the drivers of potential improvement, we investigate how the fixed duration policy and the myopic heuristic allocate available capacity among different control types.

Observation 7 (Capacity Allocation). The myopic heuristic allocates less capacity to patients in better health states and more to those in worse health states.

Figure 2 illustrates one such set of results for convex QoL and worst initial state. When capacity is very tight, the myopic heuristic generates value by significantly increasing the capacity allocation to the worst control types—worsened and unchanged—and commensurately decreasing allocation to the best types. When capacity is relatively abundant, the difference in allocation between the myopic heuristic and fixed duration policy decreases.
8.6. Improved Patient Access to Care

We further explore how the myopic heuristic achieves a more effective capacity allocation as described above. Intuitively, one might expect that compared to the fixed duration policy, the myopic heuristic results in higher frequency of visits for patients with worse health states and lower frequency for patients with better health states. Table 6 shows that this intuition is true when capacity is not very tight, as described in the observation below.

Observation 8 (Access—Large Capacity). When capacity is not very tight, the duration between visits under the myopic heuristic is larger for better health states and smaller for worse health states compared to the fixed duration policy.

This observation confirms the intuition of MCF management that the fixed duration policy results in more frequent visits for the healthier patients than might be necessary. However, the intuition needs to be modified when the capacity is very tight. In this case, the fixed duration policy fails to see a nontrivial fraction of the patients because of the rigidity in the recommended visit intervals. In contrast, the myopic heuristic accommodates these additional patients by increasing the visit interval for all patient segments.

Observation 9 (Access—Small Capacity). When capacity is very tight, the fraction of patients without any appointment is lower under myopic heuristic compared to that under the fixed duration policy.

In summary, we find that the myopic heuristic achieves near-optimal QALY benefits and has significant potential to improve access to care for MCF operations. This is particularly important because MCF had to discontinue the operation of one van due to budgetary limitations, thus making the capacity even tighter than before.

9. Conclusion and Future Work

In this paper, we analyze decisions related to the allocation of appointment slots in community-based healthcare for chronic diseases in a nonprofit setting. We develop an integrated approach to capacity allocation that incorporates a clinical model of disease progression with the operational dynamics. We formulate this decision problem as a stochastic dynamic program, where the available capacity is allocated among patients of different health states whose transitions between states are governed by discrete-time Markov chains.

We show that the optimal policy for a special case of the problem has a relatively simple structure; based on these insights, we develop a myopic heuristic that performs very well for the more general version of the problem. In deriving the structure of the optimal policy, we provide a quantitative characterization of “better” and “worse” health states in the absence of perfect observability of each patient’s health state in every period. Applying our results to data from a mobile healthcare provider shows that significant improvement in health outcomes can be obtained over the...
Table 6. Access characteristics of the myopic heuristic and the fixed duration policy.

<table>
<thead>
<tr>
<th>Capacity</th>
<th>Health state</th>
<th>Current (intended)</th>
<th>Current (actual)</th>
<th>Myopic</th>
<th>Fraction of patients not seen (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Average duration between visits</td>
<td></td>
<td></td>
<td>Current</td>
</tr>
<tr>
<td>5</td>
<td>Controlled</td>
<td>3</td>
<td>7.87</td>
<td>15.71</td>
<td>5.00</td>
</tr>
<tr>
<td></td>
<td>Improved</td>
<td>1</td>
<td>6.34</td>
<td>11.35</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Unchanged</td>
<td>1</td>
<td>6.30</td>
<td>9.06</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Worsened</td>
<td>1</td>
<td>6.18</td>
<td>6.90</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>Controlled</td>
<td>3</td>
<td>5.72</td>
<td>9.05</td>
<td>0.02</td>
</tr>
<tr>
<td></td>
<td>Improved</td>
<td>1</td>
<td>3.91</td>
<td>4.44</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Unchanged</td>
<td>1</td>
<td>3.95</td>
<td>3.28</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Worsened</td>
<td>1</td>
<td>3.96</td>
<td>2.34</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>Controlled</td>
<td>3</td>
<td>4.25</td>
<td>5.29</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>Improved</td>
<td>1</td>
<td>2.38</td>
<td>2.00</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Unchanged</td>
<td>1</td>
<td>2.42</td>
<td>1.75</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Worsened</td>
<td>1</td>
<td>2.44</td>
<td>1.47</td>
<td></td>
</tr>
</tbody>
</table>

current practice of specifying fixed duration between visits for patients of different health states. Our proposed policy achieves this improvement by flexibly adjusting the visit duration to accommodate capacity constraints and health states of the entire patient population. In addition, this flexibility also improves access; i.e., it reduces the number of patients never seen. These results indicate that significant health benefits can be obtained by improving capacity allocation decisions in community-based chronic care settings.

Our model and results provide several interesting avenues for future research. In our model, we implicitly assume that a higher level allocation of capacity between new and returning patients has been made, and hence we consider only returning patients. An extension of our model would be one that considers patients as two-dimensional constructs characterized by health state and new/returning status. This would make the patient pool size endogenous to past decisions. Results from Jones et al. (2007) suggest that the Markovian properties of disease progression hold for returning patients, but not for new patients.

We assume that all scheduled patients show up according to their appointment schedule. Another extension would be one in which a fraction of patients do not show up, leading to wasted capacity. Although some models in literature account for no-shows, none incorporate disease progression dynamics, which can influence no-show behavior (Deo et al. 2009).

The mobile care delivery model motivates an entire spectrum of operational questions that have not been explored in the literature. These include (i) joint capacity allocation between different locations and between different patient classes within a location and (ii) the assignment of mobile units to different regions of the target population. These questions are gaining importance from the perspective of global health policy because of the growing popularity of community-based healthcare delivery in developing countries. Recently, one of the authors was approached to help design a mobile healthcare system to target chronic conditions in Ghana (Culhane et al. 2010).

In conclusion, our results highlight the significant potential to improve health outcomes by making better operational decisions. This adds a new dimension to the literature on healthcare operations management that has focused primarily on the efficiency gains obtained by improving patient flow in hospitals and appointment scheduling in conventional clinics (physician offices).

Supplemental Material

Supplemental material to this paper is available at http://dx.doi.org/10.1287/opre.2013.1214.

Endnotes

1. Throughout the paper, we use boldface font to denote random scalars and random vectors.
2. The optimal policy is found through exhaustive search and does not necessarily follow the structure of Theorem 2 since Conditions (C-1) and (C-2) are not met.
3. All computations are performed on an AMD Opteron CPU core running at 2.8 GHz with 4 GB of memory.

Acknowledgments

The authors thank Stephanie Whyte, Anna Bunploog, Paul Detjen, the entire staff at the Mobile C.A.R.E. Foundation, Ruchi Gupta of Children’s Memorial Hospital and Northwestern University Feinberg School of Medicine, and Tricia Morphew of the Asthma and Allergy Foundation of America for their invaluable input on childhood asthma and mobile healthcare programs. The authors also thank Northwestern University undergraduates Thépphan Asvatanakul and Kerry Stuewer for their assistance with data analysis and collection. This research has been supported by the National Science Foundation [Grants CMII-0654398, CMII-0348622, and CMII-1131298].

References


Sarang Deo is an assistant professor of operations management at the Indian School of Business. His research is focused on...
on designing more effective healthcare delivery systems using operations management principles. His current interests include studying the economics and operations of tuberculosis diagnosis in India and HIV early infant diagnosis in sub-Saharan Africa.

Seyed Iravani is a professor of industrial engineering and management sciences at Northwestern University. His research interests are in the applications of stochastic processes, game theory, and queueing theory to the design and control of manufacturing, service operations, healthcare, supply chains, and nonprofit systems.

Tingting Jiang is a Ph.D. candidate in the Department of Industrial Engineering and Management Sciences at Northwestern University. Her main research interest lies in the optimization and game theoretical models in the healthcare sector.

Karen Smilowitz is an associate professor of industrial engineering and management sciences at Northwestern University and holds a joint appointment with the Northwestern Transportation Center. Her research interests include modeling and solution approaches for logistics and transportation systems in both commercial and nonprofit applications, working with transportation providers, logistics specialists, and a range of nonprofit organizations.

Stephen Samuelson is the president and CEO of the Frisbie Senior Center in Des Plaines, Illinois. He is the former executive director of the Mobile C.A.R.E. Foundation. In 2009, he received the Mobile Healthcare Leadership Award from the Mobile Health Clinics Network. He received an M.P.A. in not-for-profit management from Roosevelt University.