Vertically Differentiated Simultaneous Vickrey Auctions: Theory and Experimental Evidence

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We study settings where a number of sellers simultaneously offer vertically differentiated Vickrey auctions for imperfect substitute goods to unit-demand buyers. Vertical differentiation can arise from differences in item quality, item value certainty, seller reliability, or a combination of these factors. We characterize the form of the bidding equilibria and derive expressions for the corresponding allocative efficiency and expected seller revenue. When bidders are restricted to submit at most one bid, our theory predicts the existence of a unique Bayes-Nash equilibrium that resembles a form of probabilistic “mating-of-likes.” Allowing unit-demand bidders to place an arbitrary number of bids induces complex strategy profiles where bidders place positive bids in all available auctions. Higher bidder types tend to follow more targeted strategies, focusing their “serious” bids on fewer and, generally, higher quality auctions. The nature of the bidding equilibria introduces allocative inefficiencies that arise from the lack of coordination in auction selection among bidders. We test our theoretical propositions in a controlled laboratory experiment while also utilizing a domain specific risk score to help assess how the bidders’ risk type affects their bidding behavior. In support of our theory we find evidence of a probabilistic assortative matching between bidder and auction types. We also find that low risk type bidders tend to crowd on the highest auction and will pay a premium for the certainty it offers, whereas high risk type bidders fail to appropriately adjust for risk associated with the lowest auction, leading to overbidding. These lead to an interesting focal anomaly whereby bids are concentrated on the highest and lowest auctions, bypassing intermediate auctions.

Key words: competing auctions; simultaneous auctions; vertical differentiation; reputation; game theory; laboratory experiment

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1. Introduction

Recent advances in information technology have led to a proliferation of the use of auctions as a mechanism for selling products and services. As more sellers offer their goods via auctions and as lower search costs enable buyers to identify multiple sources of similar products, the process of buying a product or service increasingly reduces to the problem of bidding in a competing auctions setting.

The study of competing auctions is the focus of a growing literature in economics and management science (see §2). Competing auctions are distinguished with respect to whether the goods being offered are identical or differentiated, and also with respect to their relative timing (sequential, overlapping, simultaneous). The majority of prior work has assumed that the goods being auctioned are identical. Nevertheless, in most real-life settings, competing auctions sell similar but not identical goods; treating these goods as imperfect substitutes not only is a closer approximation to reality but also has important implications for bidding strategies and allocative efficiency.

In this paper, we augment the literature on competing auctions by analyzing settings where (a) the goods being auctioned are vertically differentiated imperfect substitutes, (b) all auctions end simultaneously, and (c) bidders have incomplete information about each other’s valuation. The following examples provide a sample of the range of real-life settings that can be modeled as simultaneous auctions of vertically differentiated goods.

- **Oil and mineral rights auctions**: Under current policies, leases that confer the right to explore for oil and minerals and develop tracts of federally controlled lands are transferred to the private sector at
periodic oral or sealed-bid auctions conducted by the Bureau of Land Management (U.S. Department of the Interior 2009). Several tracts are auctioned simultaneously. Auctioned tracts differ with respect to their relative attractiveness to bidders. For example, some tracts might be considered more likely to contain deposits of valuable minerals than others. Under the assumption that all bidders have identical information about the relative attractiveness of the auctioned land parcels (even though their absolute valuations of each parcel may differ), the setting exhibits vertical differentiation.

- **Procurement auctions:** Any sealed-bid procurement setting where a number of buyers issue independent competitive request for quotations (RFQs) and a number of contractors, each of whom can handle only one contract at any given time, are bidding for these contracts, can be modeled using the tools of this paper. Bidders in such settings often face value uncertainty arising from incompletely specified contracts, especially in information technology and knowledge-intensive settings where a full set of contingencies are difficult to enumerate ex ante. Vertical differentiation arises when, after factoring in the uncertainty, some contracts have a higher expected value than others and all bidders agree with respect to the relative ordering. Another variant of vertically differentiated procurement auctions are settings where the buyers have different reputations for following through with the RFQs. In all these cases, even if auctions do not end simultaneously, there is usually a time lag between the submission of binding bids and the announcement of winners. From the bidder’s perspective, any set of procurement auctions whose latest bid submission deadline precedes the earliest winner announcement date must be treated as simultaneous.

- **Internet auctions:** Each day eBay and other online auction marketplaces stage thousands of independent auctions for units of similar goods. For example, a search for an “iPod Nano 4GB” on any random day of 2009 was likely to reveal more than one thousand listings. No two of these listings are exactly identical: sellers offer iPods in different conditions and with different accessories; furthermore, sellers have widely varying online reputations. If one assumes that all buyers have a higher private value for an iPod that is in a better condition and that is sold by a more reputable seller, eBay can be thought of as a setting of vertically differentiated competing auctions. And even though no two auctions end at precisely the same time, in practice, it is reasonable to assume that most users do not spend their entire day watching eBay auctions. From a single user’s bidding perspective, all auctions that end in the time interval between that user’s successive visits to the eBay website must be treated as simultaneous. Alternatively, vertically differentiated auctions that end within seconds of each other, essentially depriving the losing bidder the time to rebid in a subsequent auction, would also fall under this category.

- **Grid computing auctions:** Grid computing networks are increasingly using auctions as a mechanism for allocating CPU capacity to computational jobs (Bapna et al. 2008). In most such networks, the set of available CPUs exhibits vertical differentiation (e.g., different CPUs have different speed and reliability, say, through differences in installed security patches or operating system updates). The setting, therefore, is another special case of vertically differentiated simultaneous auctions.

We attack the inherent complexity of our setting by following a three-pronged approach that encompasses game-theoretic analysis, numerical simulation, and controlled experiments.

Our game-theoretic analysis derives the form of equilibrium bidding strategies in settings where a number of sellers simultaneously offer vertically differentiated Vickrey auctions for imperfect substitute goods to unit-demand buyers. To conquer complexity, we conduct our analysis in two stages. First, we restrict bidders to submitting at most one bid and derive a unique Bayes-Nash equilibrium that resembles a stochastic version of the mating-of-likes result of Becker (1973): Auctions are ranked according to their relative attractiveness, and buyers self-select into a finite number of zones according to their types (private valuations). Buyers whose types fall in the highest zone always bid on the highest auction; buyers whose types fall in the second-highest zone randomize between the top two auctions, assigning slightly higher probability to selecting the second auction. More generally, buyers whose types fall in the kth zone randomize between the top k auctions, assigning increasingly higher probability to selecting lower auctions. This bidding behavior is a form of probabilistic positive assortative matching: bidders assess where they stand on the valuation scale and assign higher probability to bidding on the auction that “matches” their respective zone, while also occasionally “taking chances” on higher auctions.

1 eBay auctions also have a significant sequential component in that sellers constantly list additional units of popular items. A more precise way to think of eBay would therefore be as a combination of sequential rounds, each of which consists of a number of vertically differentiated simultaneous auctions. Online Appendix VI shows that the addition of the sequential dimension leads to an across the board reduction of bid amounts but does not qualitatively change the properties of the within-round bidding equilibria.

2 Roth and Ockenfels (2002) point out that last minute bids on eBay sometimes fail to go through because of transmission delays.
When we relax the single-bid constraint, we show that, even though bidders have unit demand, in equilibrium it is optimal for bidders to place nonzero bids in all auctions. The optimal bid amount in each auction is equal to the bidder’s expected valuation of the respective auction multiplied by the probability that the bidder will not receive the item from any of the other auctions, given her other bids. This specification gives rise to multiple equilibria that can only be approximated numerically. Our numerical simulations indicate that these equilibria qualitatively maintain the mating-of-likes property of the single-bid case: low bidder types tend to place equally “serious” bids (i.e., bids that are close to their private valuation) on all auctions, whereas higher bidder types tend to follow more targeted strategies, focusing their serious bids on fewer and, generally, higher auctions.

To test how well our theory describes actual bidding behavior and to obtain some insight into how risk attitudes affect behavior (to retain tractability our theoretical analysis assumes risk-neutral bidders), we follow a long tradition of controlled laboratory experiments using induced value theory (Smith 1976). Vertical differentiation across auctions was modeled by assuming that sellers sell identical goods but each seller has a different reputation, defined as the commonly known probability of shipping an item after receiving payment. Subject risk factor scores were generated using a survey developed by Weber et al. (2002). Our experimental analysis found interesting conformance as well as some departures from theory. We found statistically significant evidence of a probabilistic assortative matching between bidder and auction types: on average higher bidders were more likely to place fewer bids and to target higher auctions. This behavior was moderated by risk attitudes: We found that less risk tolerant bidders tended to focus on the top reputation sellers more often than theory would predict. At the same time, bidders targeting lower reputation (higher risk) sellers were on average more risk tolerant, and they generally failed to appropriately adjust for the associated risk of bidding.

An important result of both our theoretical and experimental analyses is that simultaneous auctions suffer from low allocative efficiency. This is an inherent drawback of decentralized auctions with incomplete information that has particularly severe consequences in settings with vertical differentiation: In the absence of complete information about each other’s private value, bidders cannot coordinate with each other when choosing where to bid. It is then possible that two or more high value bidders who could all win if they could coordinate and choose adjacent auctions will cluster on the same auction, reducing efficiency. Our work shows that the extent of such inefficiencies can be substantial. In our view, such inefficiencies lower the attractiveness of independently offering goods via auctions in settings where competitors are inclined to do the same. We argue that efficiency can be improved by centralizing such auctions and predict the emergence of market intermediaries who will assume such a coordinating role.

The rest of this paper is organized as follows. Section 2 reviews the related literature. Section 3 introduces our theoretical model. Section 4 describes our controlled laboratory experiment. Section 5 discusses managerial implications and possible directions of future work. Appendices I–VI are provided in the e-companion.3

2. Literature Review

The study of competing auctions is the focus of a growing literature in economics and management science. This literature can be structured along three important dimensions: First, according to the temporal nature of competition, competing auctions can be classified into sequential (e.g., Ashenfelter 1989, DeSilva et al. 2005, Goes et al. 2009), overlapping (e.g., Bapna et al. 2009) and simultaneous (see below). A second dimension is defined by whether competing auctions are identical or differentiated. A third dimension can be based on the information environment, differentiating studies where bidders are assumed to have private versus common values and complete versus incomplete information about each others’ valuation.

Because our work studies simultaneous auctions, we restrict the focus of this section to past work in this area. The seminal paper of Engelbrecht-Wiggens and Weber (1979) provides one of the starting points for the game-theoretic study of simultaneous auctions. The authors analyze a market where there are multiple single-unit auctions of an identical good (a dresser) and several unit-demand bidder couples who can choose to visit one auction or separate and visit two auctions. The authors derive a mixed strategy Nash equilibrium for the special case where the number of buyers is large. Subsequently, Krishna and Rosenthal (1996) study the case of simultaneous auctions with complementary goods, whereas Rosenthal and Wang (1996) further expanded this setting to the case of common value items. Szentes and Rosenthal (2003) study simultaneous auctions of perfect substitute goods, but their setting is restricted to three sellers and two bidders, each bidder having the same value (and therefore having complete information about the other bidder’s value). Peters and Severinov (2006) study simultaneous identical ascending auctions. Their main finding is that when there are no

3 An electronic companion to this paper is available as part of the online version that can be found at http://mansci.journal.informs.org/.
bidding costs and when bidders share a common fixed buying horizon (or end time), it is an equilibrium strategy for bidders to submit a bid on an auction with the lowest standing bid. If the strategy is followed by all the bidders, prices are expected to be uniform across all auctions. Gerding et al. (2008) study simultaneous Vickrey auctions offering identical goods. They characterize the form of bidding equilibria for arbitrary numbers of buyers and sellers, by imposing the assumption that a single global bidder is competing with a set of local bidders, i.e., bidders who only bid on a single, randomly chosen, auction. Bikhchandani (1999) studies simultaneous auctions of heterogeneous objects to multiunit-demand buyers where valuations are commonly known and each buyer’s value for an object may depend upon the other objects he obtains. Szentes (2007) studies two-object, two-bidder simultaneous auctions in settings where information is complete and the objects are either complements or substitutes. Byde (2001) and Bertsimas et al. (2009) propose dynamic programming bidding algorithms for a single item in an online auction, as well as for multiple items in three simultaneous or overlapping online auctions. They however “ignore the effect” (p. 23) of ratings of sellers, i.e., of seller heterogeneity, on bidder behavior. Beil and Wein (2009) study a setting of two simultaneous auctions for identical goods where some of the unit-demand buyers are dedicated to each auction while others participate in both auctions. Hoppe (2008) analyzes simultaneous auctions for a homogeneous good in an experimental setup. He observes late bidding and bid concentration on single auctions as well as coordination failures among bidders, just as we do.

Our study differs from all previous studies of simultaneous auctions in that it is the first to consider systems of vertically differentiated simultaneous auctions, i.e., auctions that offer imperfect substitute goods that can be rank ordered according to their relative attractiveness to bidders and, where all bidders agree with respect to this relative rank ordering. We argue that systems of differentiated auctions are even more common than systems of identical auctions. Furthermore, vertical differentiation has an important impact on bidding strategies compared to the case of identical auctions. Our work, thus, augments the literature in a practically relevant and nontrivial direction.

3. Theoretical Model

Our model considers $M$ sellers and $N$ risk-neutral buyers trading in a market. Each seller offers a sealed-bid, second-price auction for a single unit of an imperfect substitute good (e.g., a used unit of a given product in a variety of conditions), whereas each buyer has an inelastic demand for one unit of this good. All seller auctions begin and end simultaneously. Each buyer has a privately known type $t \in [0, 1]$ that determines her unit valuation. Buyer types are independently drawn from the same cumulative probability distribution $F(t)$ with associated density function $f(t) > 0$. Buyers know the total number of buyers $N$, their own type $t$, and the probability distribution of other buyers’ types.

We model vertical differentiation by assuming that buyer $t$’s valuation of seller $k$’s good is equal to $v_k(t) = tr_k$, where $r_k \in [0, 1]$ is publicly known and common across all buyers. In the rest of the paper, we will refer to $r_k$ as the auction’s type. A risk-neutral buyer of type $t$ who wins seller $k$’s auction at price $p$ thus expects to get surplus $tr_k - p$.

Depending on the specifics of the setting of interest, an auction’s type $r_k$ can have a number of different interpretations, including, but not limited to the following:

- **Item quality**: $r_k$ represents the ratio of every buyer’s valuation of seller $k$’s good relative to their valuation of a “perfect” (e.g., brand new) good of the same class. An example of a setting where this interpretation is pertinent are simultaneous auctions of used goods that are sold in different conditions.

- **Value uncertainty**: $r_k$ represents the (commonly agreed upon) expected value of the good relative to a “perfect” good of certain value. An example of a setting where this interpretation is pertinent are oil and mineral auctions where there is uncertainty with respect to the mineral and oil deposits associated with a given land parcel.

- **Seller reputation**: $r_k$ represents the buyers’ commonly held subjective probability that the seller will fulfill the transaction and will deliver the promised good. Common seller reputations can arise from public information about a seller’s past behavior provided by an online reputation mechanism (Dellarocas 2003) or some other information that is available to all buyers. Examples of settings where this interpretation is pertinent are Internet auctions and grid computing auctions (in the latter case $r_k$ can be thought of as a CPU’s probability of completing the job without crashing).

In the rest of the paper, we assume that an auction’s index $k$ indicates its relative rank within the set of competing auctions. Specifically, we assume that

$r_1 \geq r_2 \geq \cdots \geq r_M$.

Trade is organized in the following way. All buyers simultaneously arrive in the market and discover their types. Sellers simultaneously announce their auctions and types $r_k$. Buyers submit bids to a subset of available auctions. Each auction is won by its highest bidder who pays an amount equal to the second-highest bid.
A buyer’s decision problem has two distinct components: (i) select which auctions to bid on; (ii) decide how much to bid on each selected auction.

### 3.1. One-Bid Equilibria

To derive our initial insights, we consider the special case where each unit-demand buyer is restricted to place at most one bid. Equilibrium bidding behavior has an elegant characterization in the special case.

Because all auctions are sealed-bid, once buyers have decided on which auction to place their bid, individual auctions proceed independently. According to the theory of second-price auctions, buyer t’s optimal bid on seller k’s auction is independent and equal to her expected valuation \( v_k(t) = t_{rk} \). This observation simplifies our problem considerably as it allows a buyer’s bidding strategy to be uniquely determined by her auction selection strategy.

An auction selection strategy can be represented as a vector \( s(t) = (s_1(t), \ldots, s_M(t)) \in [0, 1]^M \) where \( s_k(t) \) denotes the probability that buyer t will bid (an amount equal to \( t_{rk} \)) on seller k’s auction. An auction selection strategy is pure if and only if all components \( s_k(t) \) are either 0 or 1. Throughout this section, we restrict our attention to strategy vectors that are piecewise continuous in buyer type t.

If buyers are restricted to select at most one auction, they will choose the auction that maximizes their expected surplus, given their beliefs about every other buyer’s strategy. Specifically, buyer t’s expected surplus from bidding her expected valuation on seller k’s auction is equal to

\[
V_k(t) = \sum_{n=0}^{N-1} \Pr[n \text{ other bidders choose auction } k] \\
\times \Pr[\text{all } n \text{ other bidders have types } \leq t] \\
\times r_k(t - E[\text{second-highest type }| \text{highest type } = t \wedge \exists n \text{ other bidders}]).
\]  (1)

Observe that the expected surplus from winning an auction increases with an auction’s type \( r_k \) but also with the expected distance between the highest and second-highest bidders’ types. Lower-type auctions that receive fewer bids may, thus, generate a higher expected surplus than higher-type auctions that receive more bids. Furthermore, the probability of winning an auction decreases with the number of bidders of similar or higher types. In selecting their strategy, buyers must, therefore, trade off the incentive to bid on higher-type auctions (increases the expected private value conditional on winning) against the incentive to bid on less popular auctions (increases the probability of winning and decreases the expected payment). The tension between these two opposing forces gives rise to the resulting equilibria.

Given a selection strategy \( s: [0, 1] \rightarrow [0, 1]^M \), the following set of functions will play an important role in the analysis:

\[
Q_k(t \mid s) = \int_0^t s_k(u) f(u) \, du, \quad k = 1, \ldots, M. \tag{2}
\]

In the rest of the paper, we will omit the dependence on s when it is implied. The quantity \( Q_k(t) \) is equal to the probability that a buyer is of type t or lower and bids on seller k’s auction. The quantity \( Q_k(1) \) then represents the probability that a randomly chosen buyer bids on seller k’s auction. The quantity \( Q_k(1) - Q_k(t) = \int_t^1 s_k(u) f(u) \, du \) is equal to the probability that a buyer is of type t or higher and bids on seller k’s auction. Finally, the quantity \( 1 - Q_k(1) + Q_k(t) \) is equal to the probability that a buyer is of type t or lower or does not bid on seller k’s auction. This, in turn, is equal to the probability that a bidder of type t will win auction k if he is competing against exactly one other bidder.

The following lemma is a generalization of standard auction theory results (McAfee and McMillan 1987, Riley and Samuelson 1981) in the context of simultaneous auctions:

**Lemma 1.** Let \( s(t) = (s_1(t), \ldots, s_M(t)) \in [0, 1]^M \) denote a buyer’s beliefs about every other buyer’s auction selection strategy and let \( Q_k(t \mid s) \) be the functions defined by those beliefs and Equation (2). The following are true:

1. The number of bidders on seller k’s auction follows a binomial subjective probability distribution with mass function:

\[
P_k(m \mid s) = \binom{N}{m} Q_k(1 \mid s)^m (1 - Q_k(1 \mid s))^{N-m}. \tag{3}
\]

2. The subjective probability that a buyer of type t who bids her expected valuation on seller k’s auction will win the auction is given by

\[
W_k(t \mid s) = (1 - Q_k(1 \mid s) + Q_k(t \mid s))^{N-1}. \tag{4}
\]

3. Buyer t’s expected surplus from bidding her expected valuation on seller k’s auction is given by

\[
V_k(t \mid s) = r_k \int_0^t (1 - Q_k(1 \mid s) + Q_k(x \mid s))^{N-1} \, dx. \tag{5}
\]

In the above expressions, the quantity \( (1 - Q_k(1 \mid s) + Q_k(t \mid s))^{N-1} \) is equal to the probability that no other bidder has type higher than t and bids on seller k’s auction.

A (Bayes-Nash) symmetric auction selection equilibrium is a strategy \( s^*: [0, 1] \rightarrow [0, 1]^M \) that maximizes the expected surplus of all buyer types subject to the

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4 Full proofs of all propositions and lemmas are given in Online Appendix I.
assumption that all buyers believe that other buyers are also following strategy $s^*$. Specifically, $s^*$ satisfies the following incentive compatibility constraints:

$$s_k^*(t) = 0 \implies V_k(t \mid s^*) \leq V_k(t \mid s^*)$$

$$0 < s_k^*(t) < 1, \forall t \in (0, 1)$$

$$s_k^*(t) = 1 \implies V_k(t \mid s^*) \geq V_k(t \mid s^*)$$

for all $t \in [0, 1]$ and $1 \leq k, l \leq M$. (6)

The one-bid restriction requires that $\sum_{k=1}^{M} s_k^*(t) = 1$ for all $t \in [0, 1]$.

Intuition suggests that bidders whose types are near the top of the distribution (and who, therefore, have a good chance of winning whichever auction they decide to bid on) will select higher auctions, whereas bidders with low types will choose lower auctions in the hope of maximizing their prospects of winning something. Interestingly, it turns out that no pure strategy auction selection equilibrium exists.

**Lemma 2.** There exists no pure strategy one-bid auction selection equilibrium.

An informal justification of the nonexistence of a pure equilibrium can be based on the following argument. Suppose that a pure strategy equilibrium exists. Then, the assumption of piecewise continuous strategies implies that this equilibrium can be defined in terms of type zones $(t_L, t_z)$ such that, all types within a zone always bid on a specific seller’s auction. Assume a pure strategy that prescribes that all buyers in zone $(0, t]$ bid on seller $k$’s auction. Sellers whose types are near zero have the lowest probability of winning. If low types find it optimal to bid on seller $k$’s auction, then higher types will find it even more so. Therefore, any pure equilibrium must have $t = 1$: all buyers bid on seller $k$’s auction. But then, any buyer who bids on another seller’s auction is guaranteed to win that auction. If there exists some other seller with positive reputation, then buyers whose types are sufficiently close to the bottom of the distribution will prefer to deviate from the pure equilibrium. Thus, no pure strategy can be an equilibrium.

The following result sheds further light into the structure of the mixed auction selection equilibria:

**Lemma 3.** All one-bid auction selection equilibria satisfy the following properties:

1. $s_k^*(t_0) = 1$ implies $s_k^*(t) = 1$ for all $t \in (t_0, 1]$.
2. $s_k^*(0) = 0$ implies $s_k^*(t) = 0$ for all $t \in (t_1, 1]$.
3. $s_k^*(0) = 0$ implies $s_k^*(0) = 0$ for all $l > k$.
4. For all $1 \leq k, l \leq M$, if there exists $t_0 \geq 0$ such that $s_k^*(t), s_l^*(t)$ both switch from positive values to zero at $t_0$ then $t_0 = t_l$. 

Lemmas 2 and 3 together with the assumption of piecewise continuous strategies allow us to provide a general characterization of auction selection equilibria. The following result directly ensues:

**Proposition 1.** All one-bid auction selection equilibria satisfy the following properties:

1. They must be mixed everywhere, except for an interval at the top of the type space.
2. Such equilibria can be characterized in terms of a sequence of type intervals:

$$(0, t_{L-1}], (t_{L-1}, t_{L-2}], \ldots, (t_z, t_{z-1}], \ldots, (t_z, 1]$$

with the property that buyers whose types fall within interval $(t_z, t_{z-1}]$ randomize among the $z$ sellers with the highest reputation.

3. The choice set (set of sellers among which buyers randomize) of a given type interval is equal to the choice set of the immediately preceding type interval minus the least reputable seller of that interval.

Figure 1 depicts the general form of these equilibria. We show that for a given set of auction types $r_k$, there exists a unique one-bid auction selection equilibrium with the above properties. Proposition 2 provides the details:

**Proposition 2.** Consider a setting where $M \geq 2$ simultaneous sealed-bid, second-price auctions with types $r_1 \geq r_2 \geq \cdots \geq r_M$, each offering one unit of an imperfect substitute good, are competing for $N \geq 2$ unit-demand buyers, independently drawn from the same type distribution $F(t)$. Define $L$ as the lowest integer $2 \leq L \leq M - 1$ for which $r_{L+1} < (1 - 1/(\sum_{i=1}^{L} \sqrt{1/r_i}))^{N-1}$. If no such integer exists, then $L = M$. The following clauses describe the properties of the unique one-bid auction selection equilibrium.

1. Buyers are divided into $L$ zones according to their type. Let $t_z, z = 0, 1, \ldots, L, t_0 = 1, t_L = 0$ denote the zone delimiters. Buyers whose types satisfy $t_z < t \leq t_{z-1}$ belong to zone $z$.
2. Zone $z$ buyers randomly choose among sellers $k = 1, \ldots, z$ with corresponding selection probabilities:

$$s_{zk} = \frac{N \sqrt{1/r_k}}{\sum_{i=1}^{z} N \sqrt{1/r_i}}.$$  

3. If $L < M$, then sellers $L+1, \ldots, M$ are never chosen by any buyer.
Zone delimiters \( t_z, z = 0, 1, \ldots, L - 1 \) are solutions of the following equation:

\[
F(t_z) = \left( \frac{N-L}{\sqrt{\sum_{i=1}^{z} \frac{N-L}{r_i}} + \sum_{i=1}^{z} \frac{N-L}{r_i}} \right) - (z - 1).
\]

The expected number of bids on seller \( k \)'s auction is equal to

\[
B_k = \begin{cases} 
N \left(1 - (L-1) \frac{1}{\sum_{i=1}^{L} \sqrt{1/r_i}} \right) & \text{if } k \leq L, \\
0 & \text{if } k > L.
\end{cases}
\]

Observe that each type zone's auction selection probabilities \( s_{zk} \) are inversely proportional to the types of auctions among which buyers randomize. Thus, lower auctions are chosen more often by buyers of a given type zone. Nevertheless, because higher auctions are considered by buyers of more type zones, the total expected number of bids \( B_k \) on an auction monotonically increases with its type.

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The aforementioned auction selection equilibria always involve some mixing: irrespective of the distribution of auction types, buyer types \( t \in (0, t_1] \), \( t = F^{-1}(N/L) \sqrt{r_2/r_1} \) (see Part 4 of Proposition 2) always randomize between at least the two highest auctions; only buyer types \( t \in (t_1, 1] \) follow a pure bidding strategy.

An interesting aspect of the auction selection equilibrium is Part 3 of Proposition 2: that, if \( r_{l+1} < (L-1)/(\sum_{i=1}^{L} \sqrt{r_i}) \), then auctions \( L+1, \ldots, M \) do not receive any bids. The condition can be equivalently expressed as

\[
r_{l+1} < \left( \frac{L-1}{L} \right)^{N-1} \left( \frac{1}{L} \sum_{i=1}^{L} (r_i)^{-1/(N-1)} \right)^{(N-1)}.
\]

The latter is a condition between auction-(\( L+1 \))'s type and the \((-1/(N-1))\)-power mean of all higher-ranked auction types. The condition is met as long as \( r_{l+1} \) is not substantially lower than the average type of higher-ranked auctions. The intuitive interpretation of this condition, thus, is that if auction types are relatively uniformly spread out in the interval \([0,1]\), then all auctions will receive some bids (buyers whose types belong to the bottom zone randomize among all auctions). If, on the other hand, there is a cluster of auctions whose types are substantially higher than those of the rest of the auctions, then it is possible that no buyer will place any bids on the lowest auctions.

3.2. Allocative Efficiency

The allocative efficiency of a system of auctions is an important market-level property because it characterizes the extent to which the market maximizes social welfare by allocating items to the buyers that value them most. In our setting, an analysis of allocative efficiency has particularly important implications because it can help market operators assess the extent to which the, mostly uncoordinated, simultaneous auctions that are increasingly encountered on both offline and online settings introduce market imperfections. Allocative inefficiencies are due to imperfections in the matching between bidders and auctions that result from the probabilistic nature of the auction selection equilibrium of Proposition 2. Specifically, the lack of coordination between bidders makes it possible that two or more high bidder types will cluster on the same auction (in which case only one of them wins, and the remaining auctions will be left to lower bidder types), whereas if these same bidders could coordinate and distribute their bids to different auctions, they would all win, increasing social welfare. The precise notion of efficiency that applies to our setting is given below:

**Definition 1.** Consider a setting where \( M \) simultaneous sealed-bid, second-price auctions with types \( r_1 \geq r_2 \geq \cdots \geq r_M \) each offering one unit of an imperfect substitute good, are competing for \( N \) unit-demand buyers. Buyers are independently drawn from the same type distribution \( F \), place at most one bid and follow a symmetric auction selection strategy \( s \). The expected allocative efficiency of the system of auctions under buyer type distribution \( F \) and strategy \( s \) is equal to

\[
\eta(F, s) = \frac{\sum_{k=1}^{M} r_k H_{1,k}(F, s)}{\sum_{k=0}^{\min(M, N)} r_k F_{N+1-k, N}},
\]

where \( H_{1,k}(F, s) \) is the highest bidder's expected type in seller \( k \)'s auction, and \( F_{N, N} \) is the expected value of the \( M \)-th-order statistic of a sample of \( N \) values independently drawn from distribution \( F \).

We adopt the usual convention that the first-order statistic is the minimum of the sample and the \( N \)-th order statistic is the maximum.
The numerator of $\eta(F, s)$ is the expected social welfare resulting from the system of simultaneous auctions. The denominator is the expected maximum social welfare, attainable if, for any randomly drawn set of $N$ buyer valuations, the highest seller is matched with the highest buyer, the second-highest seller is matched with the second-highest buyer, etc.

The following lemma gives a closed-form expression for the highest bidder’s expected type.

**Lemma 4.** Consider a setting where $M$ simultaneous sealed-bid, second-price auctions with types $r_1 \geq r_2 \geq \cdots \geq r_M$, each offering one unit of an imperfect substitute good, are competing for $N$ unit-demand buyers, independently drawn from the same type distribution $F(t)$. The highest bidder’s expected type in auction $k$ is equal to

$$H_{1,k} = 1 - \int_0^1 (1 - Q_k(1) + Q_k(t))^N dt. \quad (7)$$

There is no closed-form expression for $\eta(F, s)$. However, given $M, N$, a set of auction types $r_k$, and a buyer type distribution $F$, it can be computed in a straightforward manner from (7) and the formulas of order statistics distributions (David and Nagaraja 2003). For example, Figure 2 applies the above to compute the allocative efficiency for $M = 2, \ldots, 100$; $N = 2, \ldots, 500$; uniformly spaced seller reputations $r_k = (M - k + 1)/M$; and independent and identically distributed uniform buyer valuations. Observe that when the ratio of bidders to sellers is very low or very high, efficiency is relatively high. On the other hand, when the number of bidders and the number of sellers are comparable, efficiency is low and can fall below 70%. The extent of such inefficiencies constitutes one of the most striking features of systems of vertically differentiated simultaneous auctions and calls for the implementation of coordination mechanisms that improve the resulting efficiency. We revisit this point in §5.

### 3.3. Seller Revenue

This section considers the impact of auction type on seller revenue. If a second-price auction has type $r$, then its expected revenue is equal to $U = r \times E[\text{second-highest bidder’s type}$. We begin by considering the case of a single auction. It is well known (see, for example, Riley and Samuelson 1981) that the second-highest bidder’s expected type is a function of the type distribution and equal to

$$H_2(r) = \int_0^1 [tf(t) + F(t) - 1]F(t)^{N-1} dt. \quad (8)$$

Integrating by parts and rearranging, Equation (8) can be equivalently rewritten as

$$H_2(r) = 1 + (N - 1) \int_0^1 F(t) dt - N \int_0^1 F(t)^{N-1} dt. \quad (9)$$

The following lemma generalizes Equation (9) in the case where there are $M$ simultaneous auctions.

**Lemma 5.** Consider a setting where $M$ simultaneous sealed-bid, second-price auctions with types $r_1 \geq r_2 \geq \cdots \geq r_M$, each offering one unit of an imperfect substitute good, are competing for $M$ unit-demand buyers, independently drawn from the same type distribution $F(t)$. The second-highest bidder’s expected type in auction $k$ is equal to

$$H_{2,k}^{[r_1, \ldots, r_M]} = 1 + (N - 1) \int_0^1 (1 - Q_k(1) + Q_k(t))^N dt - N \int_0^1 (1 - Q_k(1) + Q_k(t))^{N-1} dt. \quad (10)$$

Proposition 3 characterizes the second-highest bidder’s expected type in a simultaneous auction setting.

**Proposition 3.** Consider a setting where an auction with type $r_k$ (hereafter referred to as auction $k$) competes with $M - 1$ other simultaneous, sealed-bid, second-price auctions with types $r_1 \geq \cdots \geq r_{k-1} \geq r_{k+1} \geq \cdots \geq r_M$, each offering one unit of an imperfect substitute good to $N$ unit-demand buyers, independently drawn from the same type distribution $F(t)$. Let $H_{2,k}^{[r_1, \ldots, r_M]}$ denote the expected value of the second-highest bidder’s type on auction $k$. The following statements are true:

1. $H_{2,k}^{[r_1, \ldots, r_M]}$ is an increasing function of auction $k$’s own type $r$.
2. $H_{2,k}^{[r_1, \ldots, r_M]}$ is a decreasing function of every other auction’s type.
The main implication of Proposition 3 is that competition amplifies the impact of an auction’s type on expected revenue \( U_k = r_k H_{k}^{\text{H}^k_{1},...,\text{H}^k_{M}} \). When all auctions are identical, an auction’s type affects the maximum valuation for that auction’s good (it is scaled by \( r_k \)) but does not affect the second-highest bidder’s expected type (because all buyers randomize among all auctions). On the other hand, if auctions have different types, then auctions with higher (lower) type attract more (fewer) bidders of higher valuations and end up with higher (lower) expected second-highest bidder types relative to the baseline case of identical auctions. Expected revenue is then a convex function of an auction’s type.

### 3.4. Multiple-Bid Equilibria

Most auction settings do not limit the number of bids that buyers can simultaneously place on competing auctions for similar goods. Accordingly, this section extends the preceding analysis to a setting where buyers are allowed to bid on an arbitrary number of simultaneous auctions. As before, we restrict the analysis to symmetric Bayes-Nash equilibria, that is, to equilibria where all bidders follow identical strategies. Even though all buyers have unit demand, it is plausible that some may then find it profitable to place several bids to increase their chances of winning at least one auction. On the other hand, if a buyer ends up receiving more than one item, she gets zero utility from additional units but still has to pay the respective auction prices.

When bidders are allowed to place any number of bids, individual bid amounts need no longer be equal to the bidder’s respective expected valuations. The problem becomes one of finding a bid vector \( b(t) = (b_1(t), \ldots, b_M(t)) \in [0, 1]^M \) that maximizes the bidder’s expected global surplus from the system of auctions, given her beliefs about everybody else’s bidding behavior. Let \( G_k(b_k) \) denote a bidder’s subjective probability of winning auction \( k \) conditional on placing bid \( b_k \) on that auction and also conditional on her beliefs about every other bidder’s strategy. Let \( g_k(b_k) \) denote the corresponding probability density function.

Whereas the one-bid analysis is independent of the precise interpretation of an auction’s type \( r_k \) (see §3.1), the form of a bidder’s expected surplus from placing a vector of bids depends on that interpretation. In the rest of this section, we will develop our key ideas by treating \( r_k \) as seller reputation, i.e., the probability that seller \( k \) will ship his good to the winning bidder after receiving payment. We will also assume that the only source of vertical differentiation among the competing auctions is the seller’s reputation, i.e., that these auctions otherwise offer identical goods. Online Appendix II discusses what changes if we interpret an auction’s type as its (relative) item quality.

Under the aforementioned assumptions, a bidder’s expected surplus from placing a vector of bids is given by

\[
V(t, b(t)) = t \Pr[\text{receive item from at least one seller} \mid b(t)] - \sum_{k=1}^M E[\text{payment to seller } k \mid b_k(t)],
\]

or, more formally, by

\[
V(t, b(t)) = \left[ 1 - \prod_{k=1}^M (1 - r_k G_k(b_k(t))) \right] - \sum_{k=1}^M \int_0^{b_k(t)} x g_k(x) \, dx.
\]

In the above expression, \( 1 - r_k G_k(b_k(t)) \) is the probability of not receiving an item from seller \( k \). This includes the probability of not winning seller \( k \)’s auction plus the probability of winning the auction but not receiving the item because seller \( k \) cheats. Accordingly, \( \prod_{k=1}^M (1 - r_k G_k(b_k(t))) \) is the probability of not receiving the item from any seller, and

\[
1 - \prod_{k=1}^M (1 - r_k G_k(b_k(t)))
\]

is the probability of receiving the item from at least one seller. The expression

\[
\int_0^{b_k(t)} x g_k(x) \, dx
\]

is the well-known expression for the expected payment of a single, second-price auction when bidding \( b_k(t) \). Therefore, \( \sum_{k=1}^M \int_0^{b_k(t)} x g_k(x) \, dx \) represents the buyer’s total expected costs of participating to the system of auctions.

If there are no auction participation costs, the following result shows that maximization of (12) implies nonzero bids in all available auctions for all bidder types in the interior of the type distribution.

**Proposition 4.** If bidders incur no costs for participating in multiple auctions, then for all \( t \in (0, 1) \), any bid vector \( b(t) \) that maximizes (12) must have \( b_k(t) > 0 \) for all \( k = 1, \ldots, M \).

The form of the surplus-maximizing equilibrium bid vector can be obtained by setting the partial derivatives of (12) to zero:

\[
\frac{\partial V(t, \cdot)}{\partial b_k} = g_k(b_k(t)) \left[ t r_k \prod_{j=1, j \neq k}^M (1 - r_j G_j(b_j(t))) - b_k(t) \right] = 0.
\]

If type \( t \) has positive probability density, then at any symmetric equilibrium it must be \( g_k(b_k(t)) > 0 \). To see this, recall that \( g_k(b_k(t)) \) is equal to the probability that some bidder will post a bid equal to \( b_k(t) \) on auction \( k \). If \( b_k(t) \) is part of type \( t \)’s equilibrium vector, then it
must be chosen by at least type \( t \), which implies that 
\( g_k(b(t)) > 0 \). The second part of (13) then yields

\[
b_k(t) = (r_k) \prod_{j \in \{1, \ldots, M\} \setminus \{k\}} (1 - r_j g_j(b_j(t))). \tag{14}
\]

In words, type \( t \)'s optimal bid in auction \( t \) is equal to the bidder's expected valuation \( r_k \) multiplied by the probability of not receiving the item through any of the other auctions, given the bidder's other bids \( b_j(t) \) and her probability assessment \( G_j(b_j(t)) \) of winning each auction if every other bidder follows the same bidding strategy. The reader can verify that expression (14) is also equal to the bidder's marginal utility of winning auction \( k \), given bids \( b_j(t) \) in all other auctions. It is straightforward to show that the Hessian of (12) is always negative definite, confirming that (14) indeed corresponds to a local maximum of (12).\(^6\)

Observe that, for a given set of seller reputations \( r_k \), each bid \( b_k(t) \) can be equivalently characterized by its bid-to-valuation (BTV) ratio:

\[
\beta_k(t) = \frac{b_k(t)}{r_k} = \prod_{j \in \{1, \ldots, M\} \setminus \{k\}} (1 - r_j g_j(b_j(t))). \tag{15}
\]

This alternative characterization has the advantage of mapping the range of every auction's possible bids into the unit interval. From (15) it follows that a buyer's BTV on each auction is negatively correlated with every one of her other bids. Therefore, the more a bidder focuses on winning a particular auction, the lower the bids she places on all other auctions.

The conceptual simplicity of Equation (15) is deceiving, because, at equilibrium, buyer beliefs must be consistent. This implies that the equilibrium functions \( G_j(b) \) depend on every other bidder's strategy over the entire type space. Specifically, in a symmetric Bayes-Nayes equilibrium with \( N \) bidders, it is \( G_j(b) = (\int \int \cdots \int h_j(z) dz)^{-1} \), where \( h_j(z) = \sum_{b_j(z)=a} f(x) \) is the sum of the probability densities of all types that, at equilibrium, bid \( z \) on auction \( j \).\(^7\)

In general the above recursive system of equations may have multiple solutions, corresponding to multiple equilibria. These solutions cannot be expressed in closed form. Nevertheless, the use of numerical methods allows us to derive important qualitative insights about the properties of such bidding equilibria.

Online Appendix III describes the computational method that can be used to derive numerical approximations to equilibrium bidding strategies. Figure 3 depicts approximate equilibrium BTV ratio vectors as a function of bidder type in three representative market settings. All three settings have six buyers whose types are independently drawn from a uniform distribution. Setting (a) has two sellers with reputations 100% and 90%; setting (b) has three sellers with reputations 100%, 90%, and 80%; and setting (c) has four sellers with reputations 100%, 90%, 80%, and 70%.

Figure 3(a) depicts the form of the bidding equilibrium in a setting with two sellers. We observe the following:

(1) Low bidder types \(( t < 0.3)\) place bids that are very close to their private value on both auctions. This

\(^6\) If there are several such solutions, the bidder chooses the one that corresponds to the global maximum.

\(^7\) If bidding strategies are not monotone, then it is possible that multiple types place identical bids.
follows directly from (15) since \( \lim_{t \to 0} G_t(tr_B(t)) = 0 \). Intuitively, low types have a very small chance of winning any auction; therefore it is optimal for them to take as many chances as they can.

(2) Bidder types in the range \( t \in [0.3, 0.6] \) place roughly equally serious bids (i.e., bids with high BTV ratio) on both auctions; however, as types increase, their BTV ratios begin to decline. This can be understood by noticing that, as types increase, placement of each bid creates an increasingly high probability that the corresponding auction will be won, and thus reduces the marginal utility of the other bid.

(3) Types in the range \( t \in [0.6, 0.8] \) focus on winning the second auction, i.e., they place a serious (high BTV) bid on auction 2 and a lowball bid on auction 1.

(4) The highest types \( (t > 0.8) \) focus their attention on winning the first auction, while placing a lowball bid on auction 2. Note that types very close to one place a bid almost equal to their expected valuation on auction 1 and a bid very close to zero on auction 2. They, thus, approximate the behavior of high types in the one-bid equilibrium.

The form of the bidding equilibrium in the three-seller setting has qualitatively similar properties: The lowest bidders place roughly equally serious bids everywhere. As bidder types increase, we observe a “zone” of types (bidder types around 0.4) that focuses on winning auction 3, followed by a zone \( (t \in [0.6, 0.7]) \) that focuses on winning auction 2, followed (interestingly) by a zone \( (t \in [0.7, 0.9]) \) that seems to focus on winning either auction 1 or 3. As before, the highest zone \( (t > 0.9) \) focuses on winning the highest auction.

In the four-seller setting, a similar, but even more elaborate, separation of behavior into informally defined zones occurs. As bidder types move from 0 to 1, we observe the following successive phases of bidding behavior: bidders place serious bids on all auctions, bidders focus on winning auction 3, bidders focus on winning auction 2, bidders focus on winning auction 4, bidders focus on winning auction 2, bidders focus on winning either auction 1 or auction 3, bidders focus on winning auction 1 or auction 3, bidders focus on winning either auction 1 or auction 4, bidders focus on winning auction 1.

On the basis of the previous examples, we can postulate that the form of the bidding equilibria in the multibid case has a somewhat similar flavor to the probabilistic assortative matching one-bid equilibrium. Bidders separate themselves into, informally defined, type zones (with fuzzy boundaries), according to their private valuation. Bidders in each type zone focus on winning (i.e., place high BTV bids) one (or two) auctions and place low BTV bids in the remaining auctions. As types grow, the focus tends to shift to winning higher auctions. However, as we observe in the case of the previous four sellers, there is no strict monotonicity of focus. For example, in Figure 3(c) we observe that the types that focus on winning auction 2 are lower than the types that focus on winning auction 3. Furthermore, we found that the specific sequence of such zones may change in different equilibria of the same setting. However, in all cases we have tried, our numerical simulations show that the monotonicity of auction revenue is maintained, i.e., higher reputation sellers on average generate higher expected auction revenue. For example, notice that, in Figure 3(c), whereas the types that focus on winning auction 2 are lower than the types that focus on winning auction 3, more types focus on auction 2 than on auction 3. On balance, this leaves seller 2 with higher auction revenue than seller 3.

The sum \( \phi(t) = \sum_{j=1}^M \beta_j(t) \) of a bidder’s BTV ratios on all auctions can serve as an indication of how targeted and confident a bidder is (in terms of winning the auction(s) where she places bids). A very confident unit-demand bidder would place exactly one bid equal to her expected valuation \( (\phi(t) = 1) \). At the other extreme, a bidder with a total lack of confidence would place high bids on all auctions \( (\phi(t) \approx M) \).

In the rest of the paper, we will refer to \( \phi(t) \) as a bidding strategy’s targeting factor. Figure 4 plots \( \phi(t) \) against bidder types for all three previous settings. We observe that the relationship is declining (though not strictly monotonically) in all three cases, implying that low valuation bidders are more likely to place multiple high-BTV bids on multiple auctions, whereas high valuation bidders follow increasingly targeted bidding strategies.

If participation in multiple auctions incurs costs (e.g., auction entry fees or simply the cognitive or computational costs of calculating the multibid equilibria), then placing multiple bids may no longer be optimal and bidders may then switch to single-bid equilibria. The following result provides a lower bound of participation costs above which all bidders will switch to placing only one bid.
Proposition 5. If a bidder’s cost of placing bids in each additional auction (beyond the first one) is higher than \( \max_{t \in [0, 1], j \in [1, \ldots, M]} V_j(t(1 - r_j W_j(t))) \), where

- \( W_j(t) \) is the probability that type \( t \) will win auction \( j \) at an one-bid equilibrium (Lemma 1, Part 2), and
- \( V_j(b/r_j) \) is a bidder’s expected surplus when she bids \( b \) on the highest available auction in a one-bid equilibrium setting (Lemma 1, Part 3), then the only bidding equilibrium that is incentive compatible is the one-bid equilibrium described by Proposition 2.

Proposition 5 provides a theoretical basis for interpreting some aspects of our experimental results.

4. Laboratory Experiment
To test our theoretical predictions and gain insight into how individual differences, such as risk preferences, impact decision making in this market structure, we conduct a controlled laboratory experiment designed in accordance with Vernon Smith’s (1976) induced value theory. Controlled experiments constitute a well established research method in the field of auctions; prior such experiments have provided valuable observations of bidding behavior that have become an integral part of modern auction theory (see, for example, Cox et al. 1982; Kagel and Levin 1985, 2002, 2005; Kagel et al. 1987). We design our experiment in line with this prior work.

4.1. Experimental Setting
The market structure of our experiment is one where six unit-demand bidders face four sellers. We implement vertical differentiation across auctions by assuming that sellers offer one unit of an identical good but have different publicly known probabilities (“reputations”) of fulfilling the transaction. Our subjects are MBA students at a top 20 global business school. All subjects assume the role of a bidder, whereas the seller’s role is computerized and programmed to act consistently with its assigned reputation. This means that if a bidder wins an auction from a seller with a 90% reputation score, the probability that the winner will receive their good is 0.9; however, regardless of whether or not the good “ships,” the winner must always pay for the unit won. In each round bidders may bid on as many seller auctions as they choose. However, the assumption of unit demand dictates that if a bidder wins two units, she must pay for both, even though the second unit is of no value to her. Once bidders have submitted bids, a subsequent screen indicates whether they have won the good and shows their profit in that round. The experiment lasts for 20 rounds. Bidders are randomly assigned different private values in each round. These private values are independently drawn from a uniform distribution with support in [6, 10]. Figure 5 depicts the experimental setup and shows the screen where bidders input their sealed bids. Bids are placed by entering a nonnegative amount in the chosen auction window, and then clicking the “bid now” button. Bidders may enter only one bid per seller, but are not restricted as to the number of sellers they may place bids on.

Subjects are paid in cash at the end of the experiment. Total profits for each bidder \( i \), denoted by \( \text{Profit}_i \), are calculated as follows:

\[
\text{Profit}_i = \sum_{t=1}^{20} (\text{Value}_{\text{Received},it} - \text{Price}_{\text{Paid},it}) + \text{Participation Fee},
\]

where \( \text{Value}_{\text{Received},it} \) is equal to bidder \( i \)’s private value at round \( t \) if at least one good is received by that bidder at round \( t \) or zero otherwise, \( \text{Price}_{\text{Paid},it} \) is the sum of prices paid at all auctions won by bidder \( i \) at round \( t \), and \( \text{Participation Fee} \) is the fixed amount subjects are paid for participating in the auction ($20).

Our experiment includes six identical sessions, each with a distinct group of six bidders and four computerized sellers. Each session lasted for 20 rounds. To maximize our data, we specify each round as the unit of analysis and econometrically adjust for the likelihood of correlated errors as per Ham et al. (2005). As a result, our total number of data points is

\[8\] Ham et al. (2005, pp. 182–183) discuss the treatment of the error term to account for individual specific correlated actions over time. They show that, whereas ordinary least squares estimation in panel data leads to overstating the precision of the estimates, a random effects estimation is efficient.
then multiply this binary variable by the “risk taking” score to get our final risk measure. This score captures the difference between the individual who is likely to engage in behavior he or she deems risky from one who is likely to engage in behavior he or she does not deem risky.

A copy of the survey is included in Online Appendix IV. For a more comprehensive discussion of the development, testing, and validation of the survey, we direct the reader to Weber et al. (2002).

4.3. Analysis of Results

The results of §3 allow us to make theoretical predictions regarding the form of bidding strategies, the impact of reputation on seller revenue, and the allocative efficiency of the system of simultaneous auctions that form the basis of our experiments. Online Appendix V applies our theory to our experimental setting and summarizes the predictions. This section discusses how our experimental results compare to theory in each of these areas. We pay special attention to how risk preferences affect behavior.

4.3.1. Bidding Strategies. Because our subjects were not restricted with respect to how many bids they could place, theory predicts that if multiple bidding was costless, at equilibrium everyone would bid on all four auctions. However, the analysis of §3.4 makes it clear that the computation of multibid equilibria requires considerable cognitive effort above and beyond the computation of a single-bid equilibrium (which, essentially, boils down to auction selection). Depending on the cost of such effort relative to the marginal benefit of placing additional bids (a comparison that depends on both a bidder’s private value and her idiosyncratic cognitive and risk attitudes) it might then be optimal to place only one bid or fewer than four bids.

Our experimental results conform to this observation. We find instances of bidders placing zero, one, two, three, and four bids, as summarized in Table 1. In the majority (57%) of instances, bidders place only one bid. As predicted by theory, bidders who place multiple bids appear to be scaling their bids to take into consideration the possibility of winning multiple auctions. This leads to average BTV ratios that are

<table>
<thead>
<tr>
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<tbody>
<tr>
<td>No. of simultaneous bids</td>
</tr>
<tr>
<td>--------------------------</td>
</tr>
<tr>
<td>1</td>
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<tr>
<td>2</td>
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<td>3</td>
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<td>4</td>
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720, providing sufficient power for our analysis. Prior to actual play-for-pay, bidders were given detailed instructions on the experiment’s objective and asked to answer a quiz (details of the quiz and instructions are available to future researchers from the authors upon request) on how to calculate payoffs under the different scenarios of winning (not winning) and receiving (not receiving) a single unit from participating in multiple auctions. The experimenter individually ensured that all subjects understood all questions in the quiz and gave personalized explanations to all those who did not. Three training rounds were used to familiarize subjects with the Web-based auction environment, the associated risk, and the payoff realization. We also polled subjects on their familiarity with Internet markets and found a high degree of awareness and prior participation in online auctions and e-commerce activities. Of our subjects, 75% had either bought or sold something online in the last 30 days, and the average amount spent per month for online purchases was between $50 and $100.

4.2. Risk Profiles

To retain tractability, our game-theoretic analysis assumes that bidders are risk neutral. An area where experimental methods can naturally augment our theoretical predictions is therefore the study of how individual risk preferences affect bidding behavior. We estimate the actual risk types of bidders in our experiment using a survey adapted from Weber et al. (2002). We administered this survey via the computerized interface and implemented it only at the conclusion of the experiment so as not to influence subject decisions.

Weber et al. (2002) note that risk attitudes and behaviors can differ depending on the context. They address five domains in their survey: financial decisions, health/safety, recreational, ethical, and social decisions. Because our study focuses on decisions under financial risk, we only consider financial risk domain questions when calculating the individual risk scores. There are a total of eight questions that align with the financial risk domain, and each question is measured on a 5-point Likert scale. Bidders are first asked to rate on a scale of 1 to 5 how risky they perceive a particular behavior to be (1 indicates “not very risky” and 5 indicates “very risky”). Next, bidders are asked the likelihood of whether they would engage in this same behavior. Again, a score of 5 is “very likely,” whereas a score of 1 is “not at all likely.” The result is the generation of two scores for each exemplified behavior: a risk perception score and a risk likelihood score. To calculate our final risk profile score, we indicate whether the “risk perception” score is above or below the mean score, indicating a −1 if it is below the mean and a 1 if it is above. The results of §3 allow us to make theoretical predictions regarding the form of bidding strategies, the impact of reputation on seller revenue, and the allocative efficiency of the system of simultaneous auctions that form the basis of our experiments. Online Appendix V applies our theory to our experimental setting and summarizes the predictions. This section discusses how our experimental results compare to theory in each of these areas. We pay special attention to how risk preferences affect behavior.

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<th>Table 1 Summary of Experimentally Observed Bidding Behavior</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of simultaneous bids</td>
</tr>
<tr>
<td>--------------------------</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>4</td>
</tr>
<tr>
<td>0</td>
</tr>
</tbody>
</table>
monotonically decreasing with the number of bids. There also appears to be a weakly inverse monotonic relationship between a bidder’s private value and the number of bids placed: bidders who place fewer bids on average appear to have higher private values. This, again, is broadly consistent with the notion that higher valuation bidders have a higher confidence of winning whichever auction they choose and, thus, tend to follow more targeted bidding strategies. In 22 instances, bidders did not place any bids. These were generally low, private, value bidders who apparently had low confidence of winning any auction and preferred to abstain from bidding. Such behavior is irrational if auction participation is costless, however, it is consistent with the presence of a (cognitive) auction participation cost.

Regression model R1 tests how a bidder’s risk score affects the number of bids placed, controlling for private value. Recall that a higher risk score indicates higher risk tolerance:

\[
\text{total \_#\_of\_bids}_{r} = \beta_{1}\text{private\_value}_{b,r} + \beta_{2}\text{risk\_score}_{b,r} + \epsilon. \quad (R1)
\]

Because the dependent variable is a count variable indicating the total number of bids placed in each round, we use a Poisson regression with robust errors to analyze the model. Results are shown in Table 2.

In agreement with Table 1, the regression results show a statistically significant negative relationship between private values and the total number of bids: higher bidder types tend to place fewer bids. Most importantly, however, the results show that a bidder’s risk score is positively associated with the total number of bids. This means that, for a given private value, more risk tolerant bidders tend to place more bids. This result is intuitively sound, because placing multiple bids increases the chances of winning more than one good, an outcome that is costly (and thus risky) to the unit-demand bidder.

The following sections examine the properties of instances where bidders placed either one or four bids, instances for which our theory provides crisp predictions.

Table 2 Results of Regression R1 (How a Bidder’s Risk Score Affects the Number of Bids Placed)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Standard error</th>
<th>z</th>
<th>P &gt; z</th>
</tr>
</thead>
<tbody>
<tr>
<td>constant</td>
<td>0.795</td>
<td>0.0270</td>
<td>29.44</td>
<td>0.000</td>
</tr>
<tr>
<td>private_value</td>
<td>-0.002</td>
<td>0.0006</td>
<td>-3.40</td>
<td>0.000</td>
</tr>
<tr>
<td>risk_score</td>
<td>0.192</td>
<td>0.0365</td>
<td>5.26</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Notes. Dependent variable is total \_#\_of\_bids: one, two, three, four. Analysis was conducted with a Poisson regression model with robust clustered errors to account for the correlated error structure that is assumed across multiple rounds.

Single-Bid Analysis. The analysis of §3.1 suggests that, when placing only one bid, bidders will choose an auction that is positively correlated with their private type and will always bid their corresponding expected valuation (i.e., the BTV ratio will always be one). More specifically, the highest (zone 1) bidder types will always choose the highest auction, whereas lower bidder types will randomize across an increasingly broad set of auctions.

Figure 6 allows us to assess the extent to which the experimentally observed auction selection behavior conforms to theoretical predictions. The figure consists of four panels; each panel shows a histogram depicting the empirical distribution of sellers chosen by buyers whose private values fall in each of the four type zones predicted by theory (see Online Appendix V for precise calculations). We observe that zone 1 and zone 4 bidders generally adhere to the probabilistic matching equilibrium of Proposition 2: With very few exceptions, zone 1 buyers choose the top reputation seller, whereas zone 4 (the lowest type) bidders randomize between all four sellers. On the other hand, whereas theory would predict zone 2 and zone 3 bidders to randomize between the top two and top three reputation sellers, respectively, it appears that they randomize between all four sellers, assigning substantially higher probability on selecting the high reputation seller.

Table 3 provides a different lens on these one-bid results, comparing the experimentally observed proportions of selecting each seller against theoretical predictions and also listing the corresponding average BTV ratios (that theory predicts would be exactly equal to 1 in all cases).

Table 3 suggests a potentially interesting “focal” anomaly, whereby bidders place disproportionate focus on the highest and lowest sellers and “bypass” the second and third sellers. Consistent with theory, the highest bidder types are more likely to place a bid on the highest seller’s auction, whereas lower bidder types are more likely to target the lowest reputation seller. Departing from theory, we observe a higher
than predicted proportion of bids (37% versus 28%) on
the highest reputation seller as well as an irrationally
high BTV ratio on both the high and low reputation
sellers.

Overbidding on the 100% reputation (“safe”) seller
where no risk adjustment is required is not overly sur-
prising; such behavior has been well documented in
laboratory experiments (Kagel et al. 1987, 1993). The
“flocking” of bidders on the most attractive among a
set of competing auction has also been documented in
overlapping auction settings (Simonsohn and Ariely
2008). On the other hand, the bypassing of middle
sellers in favor of the lowest seller is, to the best of
our knowledge, a new experimental finding.

A partial explanation for these results can emerge
if we examine how risk attitudes affect auction selec-
tion. Specifically, regression model R2 shows that,
after we control for private value, a bidder’s risk score
exhibits a statistically significant negative relationship
with the reputation of the seller where she chooses
to bid: more risk tolerant bidders choose lower rep-


umerous sellers with higher probability. The regression
model and results are shown below and in Table 4:

\[
\text{Seller_Reputation}_{b,r} = \beta_1 \text{private_value}_{b,r} + \beta_2 \text{risk_score}_{b,r} + \epsilon. \tag{R2}
\]

Based on the above results, we postulate that low
risk types flock to the high reputation seller, pay-
ing a premium for the certainty he offers, whereas
high risk types place more bids on the low reputation
seller, hoping to increase their chances of winning,
while underestimating the risk associated with win-
ning from that seller. Such biases are consistent with
prior literature on decision making under risk (Weber
and Camerer 1987, Fishburn 1988). For example, it
is often the case that when individuals chose among
riskier alternatives, the weight they assign to each out-
come may not correspond to the actual probability
of that outcome (Wu and Gonzalez 1996). It could
be the case that subjects bidding on the low reputa-
tion sellers overestimate the probability of actually
receiving the good they purchased, particularly when
comparing this to the certain payoff of zero if no
bid is placed. Similarly, overweighting the probability
of losses from lower reputation sellers could explain
why we observed a greater than expected number
of serious bids placed in the 100% reputation sellers’
auctions.

Multibid Analysis. The main insight of §3.4 is that,
when bidders place multiple bids, higher bidder types
tend to follow more targeted strategies, focusing their
serious (i.e., high-BTV) bids on fewer and, generally,
higher auctions. In this section, we show that these core predictions are consistent with our experimental data.

In §3.4, we defined a bidder’s targeting factor, equal to the sum of a bidder’s BTV ratios, and used it as a measure of how targeted a bidding strategy is. We also referred to the bid with the highest BTV as the serious bid, i.e., the bid placed on the auction that the bidder is most interested in winning.

Regression model R3 tests the relationship between a bidder’s private value and the bidder’s targeting factor. We include the risk score variable as a control. Results are shown in Table 5.

\[
\text{Targeting\_Factor}_{b,r} = \beta_1 \text{private\_value}_{b,r} + \beta_2 \text{risk\_score}_{b,r} + \varepsilon. \quad (R3)
\]

We observe a statistically significant negative relationship between targeting factor and private value, suggesting that higher types follow more targeted strategies, i.e., strategies that place fewer high BTV bids. This result is consistent with the results of §3.4, especially Figure 4. Interestingly, we find that higher risk scores are positively correlated with higher targeting factors. This means that controlling for private value, more risk tolerant bidders are more likely to use less targeted strategies.

Next, we analyze the relationship between the reputation of the seller where a bidder places her focal (i.e., highest-BTV ratio) bid and the bidder’s characteristics (private value and risk score). The dependent variable is ranked and can assume a value of one through four, where one is the lowest reputation seller (70%) and four is the highest reputation seller (100%). An interaction term is also included to test the relationship of bidder type and risk profile on the focal bid choice. The regression model and results are shown below and in Table 6:

\[
\text{Focal\_Seller}_{b,r} = \beta_1 \text{private\_value}_{b,r} + \beta_2 \text{risk\_score}_{b,r} + \beta_3 \text{private\_value} \times \text{risk\_score}_{b,r} + \varepsilon. \quad (R4)
\]

We find that private values have a positive association with the focal seller, indicating that average risk profile bidders are more likely to bid on high reputation sellers as private values increase. The negative and significant coefficient on the interaction term tells us this relationship is mediated by the individual’s risk propensity; as risk values decrease so does the positive relationship between private value and the focal bid reputation. Although the main effect of risk score is negative as intuition would suggest, absent the presence of private value, the coefficient is not significant.

To summarize our findings regarding bidding strategy, we find some conformance as well as departure from our analytical predictions. In accordance to our predictions, the highest value bidders are more likely to bid on the highest reputation seller. As private values decrease, the propensity to place more bids increases, as does the likelihood that bids will be placed on lower reputation sellers. More risk tolerant bidders are more likely to place more bids, as well as to bid on lower reputation sellers. When bidders place one bid, they are disproportionately more likely to choose the highest reputation seller and they fail to fully adjust for the risk of bidding on the lower reputation seller.

\subsection{4.3.2. Economic Outcomes}

Next, we analyze seller revenue, buyer surplus, and allocative efficiency as a function of bidder and seller type.

\textit{Seller Revenue.} Our theory predicts that in simultaneous auction settings where sellers have different reputations, seller revenue will be an increasing convex function of reputation. In one-bid equilibria, average seller revenue can be computed on the basis of Equation (10). In multibid equilibria, one must resort to numerical methods. In Online Appendix V, we compute the expected seller revenue for the setting of our experiments. Table 7 lists these predictions together with the actual experimental revenue average across all rounds and treatments. Observe that auction revenue is generally higher in the multibid equilibrium because the higher aggregate number of bids leads to more competition among buyers.
Table 7  Seller Revenue Theory vs. Experiment

<table>
<thead>
<tr>
<th>Seller reputation</th>
<th>Predicted average seller revenue (one-bid equilibria)</th>
<th>Predicted average seller revenue (multibid equilibria)</th>
<th>Observed average seller revenue</th>
</tr>
</thead>
<tbody>
<tr>
<td>70</td>
<td>4.57</td>
<td>4.95</td>
<td>4.79</td>
</tr>
<tr>
<td>80</td>
<td>5.35</td>
<td>5.72</td>
<td>4.75</td>
</tr>
<tr>
<td>90</td>
<td>6.15</td>
<td>6.48</td>
<td>4.99</td>
</tr>
<tr>
<td>100</td>
<td>6.98</td>
<td>7.33</td>
<td>7.08</td>
</tr>
</tbody>
</table>

Two observations stand out from Table 7. First, that the theoretical and experimental revenues are remarkably consistent for the two extreme (70% and 100% reputation) sellers. The observed values lie between those predicted under the assumption that bidders place only one bid and the assumption that bidders place bids in all auctions. This is intuitive because in our setting we observe about half the bidders placing one bid and the rest placing multiple bids. Second, we observe that seller revenue of the two middle sellers is substantially below what theory would predict. This is a direct consequence of the focal anomaly that we have previously identified: when bidders place one bid they seem to “bypass” the two middle sellers, focusing their attention on the top and bottom sellers.

**Bidder Surplus.** Online Appendix V computes expected bidder surplus as a function of private value for both the one-bid and multibid cases. Figure 7 plots these curves against the trendline that was computed from fitting a linear regression model to our experimentally observed bidder surplus data. The consistency between theory and experimental data is notable. In our experimental data, high bidders on average get a slightly lower surplus than what theory would predict, whereas low bidders get a slightly higher surplus. We attribute this slight discrepancy to the focal anomaly we have previously identified.

Because of flocking on the highest seller, high buyer types, who tend to focus on that auction, end up paying higher prices. At the same time, the bypassing of the middle sellers results in lower auction prices on those sellers, which typically benefit lower valuation buyers.

**Allocative Efficiency.** As previously discussed, simultaneous auctions are inherently inefficient because, in the absence of complete information about each other’s private value, bidders cannot coordinate with each other when choosing where to bid. Online Appendix V calculates the theoretically predicted allocative efficiency for both the one-bid and multibid cases. Figure 8 compares these theoretical predictions against the experimentally observed average efficiency levels in the 20 rounds of our experiment (averaged over all repetitions). For easier comparison with the results of §3.2, private valuations have been normalized to lie in the range [0, 1]. Once again, we observe remarkable consistency with the predictions of our theoretical models. We also observe that the linear trendline that was the result of fitting our data (6 repetitions × 20 rounds = 120 points) to a linear regression model, exhibits a statistically significant upward trend as rounds progress. We view this as evidence of bidder learning.

In summary, our analysis of economic outcomes shows remarkable consistency between theory and experimental observations. It further reinforces the extent of allocative inefficiency that is associated with systems of vertically differentiated simultaneous auctions.

![Figure 8 Allocative Efficiency Trend over Time (Private Valuations Have Been Normalized to [0, 1])](image)

*Note.* Slope of trend line is statistically significant ($p = 0.04$).
5. Managerial Implications and Research Opportunities

A lot of auction theory assumes that auctions take place in isolation. However, as technological advances continue to reduce the cost of auction setup and bidding, an increasing number of goods and services are independently offered by their sellers via auctions. This results in settings where several auctions offering imperfect substitute goods compete for bidders. Such settings are relatively new and their properties unfamiliar to both sellers and bidders. This paper analyzes bidding behavior and economic outcomes in a stylized competing auctions setting where (a) auctions are vertically differentiated, i.e., there is consensus among bidders with respect to their relative attractiveness; (b) auctions are simultaneous; and (c) bidders have incomplete information about each other’s private valuation. As discussed in the introduction, our stylized model is a reasonable approximation of a wide range of online and offline settings, including oil and mineral auctions, procurement auctions, Internet business-to-consumer auctions, and grid computing CPU allocation auctions.

Our main finding is that such settings induce probabilistic mating-of-like bidding equilibria that incur substantial allocative inefficiencies. Bidders rank-order auctions with respect to their relative attractiveness. They further assess where they stand on the valuation type space relative to other bidders and assign higher probability to focusing their efforts on winning the auction that “matches” their type, while also occasionally bidding on “higher” auctions. If bidders are restricted to place at most one bid, such behavior results in a unique Bayes-Nash equilibrium with a closed-form solution: Buyers self-select into a finite number of zones according to their types. Buyers whose types fall in the highest zone always bid on the highest auction; buyers whose types fall in the second-highest zone randomize between the top two auctions, assigning slightly higher probability to selecting the second auction. More generally, buyers whose types fall in the kth zone randomize between the top k auctions, assigning increasingly higher probability to selecting lower auctions. If bidders are not restricted to place one bid, bidding equilibria are substantially more complex but maintain the same property: low bidder types tend to place equally serious bids on all auctions, whereas higher bidder types tend to follow more targeted strategies, focusing their serious bids on fewer and, generally, higher auctions.

The probabilistic nature of these equilibria introduces substantial allocative inefficiencies. Specifically, the lack of coordination between bidders makes it possible that two or more high bidder types will place serious bids on the same auction, whereas if these same bidders could coordinate and distribute their bids to different auctions, they would all win, increasing social welfare. Our experimental data shows that these coordination failures are exacerbated by bidders’ tendency to crowd on the top (e.g., highest reputation) auction, while also failing to adjust for the risk associated with bidding on the lowest auction.

The above inefficiencies constitute the “price of anarchy” of uncoordinated competing auction markets. In our view, such inefficiencies eliminate a lot of the advantages of using auctions in settings with competition. In theory, it is possible to solve this problem completely by centralizing all the simultaneously occurring single-item auctions into a single, multi-item Vickrey-Clarke-Groves auction and asking bidders to submit menus of bids for any subset of the available items. In settings, such as eBay or oil and mineral auctions, where all competing auctions are administered by the same operator, this might be a sensible suggestion for auction marketplace operators to consider. In other settings, we anticipate that there are market opportunities for new classes of intermediaries to emerge and play a coordinating role that will increase efficiency. Our results show that this is a potentially fruitful area of both further research and entrepreneurial activity.

We expect our study to fuel future work in several areas. First, our study assumes that there is a consensus among bidders with respect to the relative attractiveness of the competing auctions. This is arguably a stylized assumption that is not likely to hold in all settings. It will be interesting to extend the ideas of this paper by analyzing settings where such consensus is not perfect, i.e., settings that combine vertical and horizontal differentiation. Second, we study sealed-bid, second-price auctions. It will be interesting to see whether ascending or descending auctions do a better job in terms of reducing the coordination issues we have identified in our setting. Third, we assume that all auctions end simultaneously. In practice, uncoordinated auctions tend to be overlapping (Bapna et al. 2009). Extending our results in settings with vertically differentiated overlapping auctions is a worthwhile avenue of future work. Fourth, introducing bidding budget constraints will be useful in modeling some interesting real-life examples of competing auctions, such as keyword auctions in sponsored search.

6. Electronic Companion

An electronic companion to this paper is available as part of the online version that can be found at http://mansci.journal.informs.org/.

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References


