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Threshold Incentives and the Sales Hockey Stick

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In this paper we study threshold-based sales-force incentives and their impact on a dealer's optimal effort. A phenomenon, observed in practice, is that the dealer exerts a large effort towards the end of the incentive period to boost sales and reach the threshold to make additional profits. In the literature, the resulting last period sales spike, is sometimes called the hockey stick phenomenon (HSP.) We show that lack of information leads to the HSP and characterize its form over multiple time periods. Under perfect information it is possible to completely eliminate the HSP, however, this may be difficult in practice. We show that the manufacturer can control the HSP by using imperfect information to set the threshold and delay its computation until the last period. We discuss an implementation plan that allows the manufacturer to do so. We then study the impact of competition on the HSP and show conditions under which the HSP can be dampened or exacerbated. We also characterize the variance of the total sales across all the periods and demonstrate conditions under which offering a bonus contract may be beneficial in controlling the variance.

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1. Introduction

Many firms offer quota-based sales-force compensation plans to motivate their sales-force to work harder and increase sales. Typically, under such sales-force compensation plans the dealer (salesperson) is paid an additional amount per unit when sales exceed a threshold[1] (sales quota) value; sometimes, instead of an additional per unit (marginal) payment, a fixed bonus is offered if the total sales figure exceeds the threshold. One example of a threshold-based sales-force incentive mechanism is the so called stair-step incentive offered by auto-manufacturer Chrysler to it's dealers

[1] We use the terms threshold and quota interchangeably throughout this paper.

(Post-Distpatch 2001). Under this incentive plan[2], Chrysler gave dealers cash based on a percent of the monthly vehicle sales target met. The bonus based incentive plan, which is the other form of such threshold-based incentives, is also quite popular in industries such as electronics and retail. Joseph and Kalwani (1998) provide several examples and discuss how bonus payments have helped firms direct sales-force effort towards specific organizational goals and increased productivity.

However, when such threshold-based (or quota-based) incentives are offered over multiple time periods, they typically result in a steep rise in sales (and sales effort) towards the end of the incentive period (Chen 2000). This rise in sales is sometimes referred to as the "*hockey stick phenomenon*" (HSP) in the literature. For example, a manufacturer may witness a sudden rise in sales during the last few weeks of the quarter if she offers the dealer a threshold incentive over a quarter. The dealer's sales pattern causes substantial difficulty in production planning and inventory management, thus increasing a manufacturer's operational cost. As is well known in the Operations Management literature, higher variability in the demand process affects over-stocking and under-stocking costs.

While providing an incentive to enhance sales is essential to increase profits, the manufacturing firm can significantly benefit if it can induce its salespeople to exert selling effort in a way that actually smoothes the demand process. As it turns out, this is possible, and the resulting benefits to the firm can be substantial. In this paper, our objective is to understand conditions that lead to the HSP and identify mechanisms that can reduce the sales spike and the sales variance. The theoretical literature, in Economics and Marketing, provides evidence that threshold-based incentives are effective (and optimal in some cases) in increasing a manufacturer's expected sales while motivating the dealer to exert effort appropriately (Sappington 1983, Basu et al. 1985, Kim 1997, Oyer 2000). However, many of these models are single time period models and hence do not include inter-temporal effects of such incentive schemes. More recently, literature in Operations Management has focused on the operational impact of such quota-based contracts (Chen 2000) and suggested strategies to smoothen the demand process. In the model suggested by Chen (2000), the HSP observed is due to the fact that dealer doesn't experience any cost of waiting. In our setting we show that even when waiting is expensive for the dealer, the HSP is still observable due to lack of perfect information.

The goal of this paper is to address the following questions: (1) how is the dealer's optimal effort characterized when there is a cost associated with waiting and when is the sales HSP observed?, (2)

[2] A dealer got no additional cash for sales below 75% of the sales target, $150 per vehicle for sales between 75.1% and 99.9% of the sales target, $250 per vehicle for sales between 100% and 109.9%, and $500 per vehicle for reaching 110% of the sales target.

how can the manufacture use additional information to choose incentive parameters appropriately and dampen the sales HSP?, and (3) how is the sales HSP affected when similar incentives are offered to multiple dealers competing with each other? We focus on the information asymmetry between the manufacturer and the dealer and the manufacturer's inability to set an appropriate threshold level to control the HSP. To characterize the sales HSP, we consider two forms of the quota-based contract: (1) when the manufacturer makes an additional marginal payment on exceeding the threshold, and (2) when the dealer makes a bonus payout on exceeding the threshold. First, we argue that offering a threshold-based incentive is beneficial to a manufacturer to enhance her expected profit. Specifically, we show that at optimality, a threshold-based contract provides a higher expected profit for the manufacturer when compared with a constant-margin contract. However, the manufacturer may experience a higher sales variance, resulting in the sales HSP, with a threshold-based contract. To understand the sales HSP, we characterize the dealer's optimal response function for both the contractual forms by formulating the dealer's problem as a stochastic dynamic program. We show that the optimal expected effort levels form a submartingale. Essentially, we show that the optimal expected effort is stochastically increasing over time. We also characterize the conditions when such an effort pattern results in the sales HSP and which of the above two contracts results in a higher sales variance. We also study conditions when the HSP is reversed.

Next, we demonstrate that the lack of information about the underlying market demand signal results in the manifestation of the HSP. A recent automotive news article (see Wernle 2006) pointed out that some Chrysler dealers complained that monthly sales targets aren't based on market demand but "on the amount of cars Chrysler has jammed down the regional business center's throat." Such anecdotal evidence suggests that a manufacturer must choose a threshold level that aligns the dealer incentives (sales effort) appropriately. We argue that by observing additional business variables (such as some macro-economic market indicator variables) correlated to the underlying demand signal, the manufacturer is able to smoothen the demand process. Essentially, linking the threshold to a correlated variable allows its computation to be delayed until the last selling period and also captures the variance of the underlying demand signal. When the correlation is perfect, we show that the HSP disappears completely. Furthermore, as long as there is some positive correlation, we demonstrate that there is a reduction in the hockey stick phenomenon. Thus, a manufacturer can simultaneously control the sales HSP and induce higher expected profits by choosing the contracting parameters appropriately.

Finally, we look at the effect of dealer competition on the sales HSP. Competition affects a dealer's sales as follows. When competition intensifies it may reduce the impact of a dealer's sales effort. However, if a dealer gains additional sales because of a competitor's sales effort, it could result in higher sales. We model both these situations and study the gap between the expected dealer efforts in a two period model. We characterize the worst-case gap and show that when competition intensifies the sales HSP is diminished. However, when competition is beneficial the HSP may be exacerbated.

The rest of paper is organized as follows. First, we provide a literature review in Section 2. We describe the model and related assumptions in Section 3. We discuss why threshold-based contracts are offered and characterize the optimal incentive parameters in Section 4. In Section 5 we characterize the dealer's response for both forms of the incentive contract. We present the value of better information in setting the threshold level in Section 6 and discuss an implementation plan in Section 6.3. In Section 7 we discuss how dealer competition affects the sales HSP. Finally, we conclude our paper in Section 8. The proofs are provided in the Appendix.

2. Literature Review

The Economics and Marketing literature on sales-force compensation and sales effort is vast. Basu et al. (1985) study the sales-force compensation plan for a single period and show that the optimal compensation plan is an increasing non-linear function of the sales (BLSS plan). Basu et al. (1985) also suggest that, for ease of implementation, a piece-wise linear approximation scheme could be used in practice. Such a scheme closely resembles the threshold-based incentive studied in this paper. The analytical approach in Basu et al. (1985) follows the moral hazard and agency theoretic framework adopted in Harris and Raviv (1979) and Holmström (1979). Raju and Srinivasan (1996) compare the BLSS plan to a simple quota-based plan (fixed pay plus a commission rate) and show, through numerical experiments, that their simple approximation results in minimal loss of optimality. The significant advantage, as argued earlier, is the simplicity of implementation in practice. Lal and Srinivasan (1993) find that a linear compensation scheme may be optimal, however such a scheme seems impractical since most sales-force incentive schemes in practice are non-linear. In a related paper Oyer (2000) shows that a quota-based plan with a bonus payment is optimal when the sales distribution function is assumed to have an increasing hazard rate. Through field experiments, Joseph and Kalwani (1998) demonstrate the growing importance of bonus payments in aligning sales incentives and increasing productivity in a wide variety of firms. In other related work Kim (1997) studies bonus contracts with limited liability and shows that a bonus contract achieves the first-best outcome. Similar to Oyer (2000), Kim (1997) assumes that the dealer chooses an effort level and then observes the private information (for example the underlying demand signal.) On the other hand, Sappington (1983) assumes that the dealer chooses his effort level after observing his private information. In this case the first-best outcome is not achievable. In this paper, we assume a model similar to the model described in Oyer (2000) where the dealer chooses his optimal effort level and then observes his private demand signal.

Detailed reviews on the sales-force management literature can be found in Coughlan and Subrata (1989) and Coughlan (1993). Most of the aforementioned literature focuses on a single period setting. However, the model discussed in this paper is multi-period model and captures inter-temporal effects of sales effort.

In the Operations Management literature, Porteus and Whang (1991) study coordination problems between one manufacturing manager and several product managers. The product managers make sales effort decisions while the manufacturing manager makes capacity and inventory decisions. Their work contributes significantly to a new stream of literature in the marketing-operations interface by linking sales-force incentives to manufacturing incentives. Bradley and Arntzen (1999) develop an aggregate planning model to include the strategic issue of capacity investment and evaluate the capacity-inventory trade-off. They also discuss how demand variability can be examined using this model. Their model also addresses investment in capacity or holding inventory.

Chen (2000) discusses the impact of sales-force incentives (over multiple time periods) on a manufacturing firm's production and inventory decisions and proposes a moving-window plan to induce salespeople to exert selling effort to smoothen the demand process. Chen (2000) assumes a linear impact of effort so that the dealer can exert all the effort in the last period creating a sales spike. In this paper we focus our attention on characterizing the dealer's optimal response when the there is a cost associated with waiting. In a following paper Chen (2005) relates sales-force compensation to the manufacturing firm's production and inventory costs and compares Gonik's scheme with a menu of linear contracts. Gonik (1978) proposed a scheme to extract maximum effort from the sales-force and induce them to truthfully reveal the underlying demand by forecasting accurately.

Other related research, in Operations Management, focuses on information sharing, surge in ordering levels in a supply chain, and the so called "bullwhip" effect. One of the reasons why the bullwhip effect is observed in supply chains is because of the lack of information flow (visibility) and misaligned incentives in a supply chain. The effect of the HSP, resulting in the manufacturer facing higher variance, is similar to the "bull whip effect" experienced by manufacturers. The operational causes for this problem, and several counter measures to mitigate the ill effects, of the bullwhip are discussed in Lee et al. (1997). Bourland et al. (1996) and Gavirneni et al. (1999) show that information sharing mitigates the "bull whip effect". Another significant cause of the "bull whip effect" is the lack of coordination mechanisms within the supply chain. Chen (2003) provides a detailed review of the literature on information sharing in supply chains. The other stream of research in this area focuses on the behavioral aspects. Sterman (2006) identifies the hockey stick phenomenon as a behavioral phenomenon resulting in supply chain instability.

Researchers have also studied effects of information sharing and alignment in terms of incentives and contract design. Related literature includes Taylor (2002), where the author studies target rebates and coordination issues, and Krishnan et al. (2004), where the authors discuss the concern of contract-induced moral hazard which arises when the dealer's effort decision is made after observing the initial sales. Cachon and Lariviere (2005) discuss situations where revenue sharing contracts do not coordinate a supply chain and develop a variation that performs better. There are other studies in the supply chain contracting literature as well using the principal agent theory. Tsay (1999) studies quantity flexible contracts and their impact on supply chain coordination. Cachon (2003) provides an excellent review of the contracting literature.

In this paper we do not consider design of optimal contracts. Our primary contribution is an explanation as to why the HSP is observed when threshold incentives are offered and how should the manufacturer adjust the parameters, specifically the threshold levels, to mitigate the sales HSP. We characterize the dealer's optimal expected effort over multiple time periods. We also examine the role of information, through the use of a correlated market indicator signal, and dealer competition in mitigating the sales HSP.

3. Model Assumptions

To investigate the HSP we consider a setting where a single manufacturer sells her product through a single dealer. The selling price of the product is fixed, i.e., the dealer is not empowered to give price discounts to customers. Thus, the only way for the dealer to grow sales is to exert effort. For example, the dealer could expend selling effort through advertising. Since the retail price is held fixed, the dealer makes a standard margin (excluding cost of effort) p for every unit sold. However, to a limited extent, the dealer's ability to increase sales by lowering the retail price (and decreasing his standard margin) is approximated by the additional cost of exerting sales effort to increase sales.

The threshold incentive is offered over T time periods, i.e., the incentive horizon comprises of T time periods. In any time period, t $(t = 1, \ldots, T)$, the dealer chooses a selling effort e_t resulting in a sales growth of $g(e_t)$, where the impact-of-effort function, $g(\cdot)$ is positive, differentiable, and concave. Therefore, the marginal impact of sales effort is diminishing. The sale, S_t, in period t is assumed to be

$$S_t = g(e_t) + X_t \tag{1}$$

where X_t is the random demand (shock) observed in period t. We assume that the market demand signal, X_t, follows a continuous, and twice differential, cumulative distribution function, F, with a probability density function f such that $f(x_t) = 0$ for all $x_t < 0$ and $f(x_t) > 0$ for $x_t \geq 0$. We also assume that the random market (demand) signals are iid. However, some results hold even if this assumption is relaxed. The distribution of the market demand signal is common knowledge (publicly known.) Such a sales-response function, as shown in Eq. (1), has also been used by Lal and Staelin (1986), Oyer (2000), and Lal and Srinivasan (1993). The total sales across all the T periods is defined by

$$S = \sum_{t=1}^{T} S_t \tag{2}$$

and the sales up to any period t is defined by

$$D_t = \sum_{j=1}^{t-1} s_j. \tag{3}$$

Let the function $h(\cdot)$ be defined as follows:

$$h(\cdot) \equiv \frac{1}{g'}(\cdot). \tag{4}$$

To explain the sales HSP, we make the following assumption:

Assumption 1. *The function $h(\cdot)$ is increasing and concave.*

Assumption 1 is not restrictive and is easily satisfied by any concave polynomial function $g(e_t) \equiv e_t^{\alpha}$, where $0 < \alpha < 1$. The dealer's cost of exerting effort is assumed to be linear in his effort, i.e.,

$$v(e_t) = \beta\, e_t \tag{5}$$

The threshold incentive is organized as follows: For every additional unit sold above the threshold, K, the manufacturer pays the dealer an amount Δ in addition to his standard margin p. Thus, the dealer's margin (excluding the cost of effort) increases to $p + \Delta$ for every unit sold above the threshold K. We refer to this contract as the "Δ-contract". In case of a bonus contract, instead of the additional marginal payment, Δ, the manufacturer offers a fixed bonus, $D > 0$, if the total sales, S, reaches, or exceeds, the threshold K. We refer to the bonus contract as the "D-contract" and consider it separately. The incentives offered are represented by their defining parameters, i.e., (p, Δ, K) represents the Δ-contract and (p, D, K) represents the D-contract.

The dealer's objective is to maximize his profit which is a function of the effort exerted, the incentive parameters, and the profit margin. The sequence of events unfold as follows. Prior to the first time period, i.e., at $t = 0$, the manufacturer offers the threshold incentive. In every time period, t $(t = 1, \ldots, T)$, the dealer decides his optimal effort level e_t^* and observes the random demand signal X_t. The dealer orders S_t units from the manufacturer which also allows the manufacturer to observe the period sale. The manufacturer keeps finished-goods inventory and is responsible for replenishing the dealer's inventory. After the last period included in the incentive horizon, i.e., when $t = T + 1$, the payoffs are determined. Figure 1 shows the sequence of events with the incentive horizon comprising of two time periods. The manufacturer's objective is to maximize her long-run

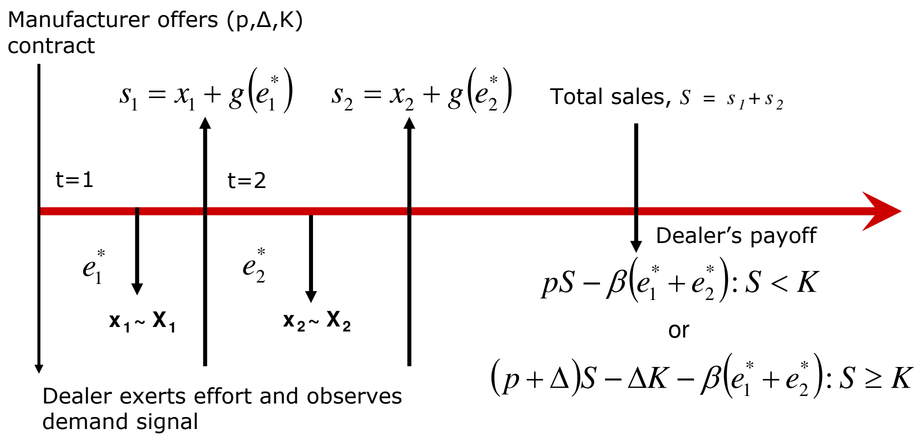

Figure 1 The sequence of events when $T = 2$.

average profits, i.e., to minimize her operational costs and the sales-force compensation payout, subject to the dealer's incentive compatibility constraint. For the remainder of the paper we use the following definitions to characterize the sales HSP when the threshold-based incentive horizon spans multiple time periods:

DEFINITION 1. The sales HSP is defined by a sales pattern such that the expected sales in a period, $\mathbb{E}[S_t]$, increases as t increases, i.e., $\mathbb{E}[S_t] \leq \mathbb{E}[s_{t+1}]$.

The operator, $\mathbb{E}$, represents the expectation over the random variable X_t[3]. However, in some instances the expected sales may decrease over time in spite of higher dealer effort. We define this situation as the reverse hockey stick phenomenon.

[3] Sometimes we use the subscript X_t for convenient exposition.

Definition 2. The sales HSP is reversed when the sales pattern is such that the expected sales in a period, $\mathbb{E}[S_t]$, decreases as t increases, i.e., $\mathbb{E}[S_t] \geq \mathbb{E}[s_{t+1}]$.

4. Should the Manufacturer Offer a Δ-Contract?

An important question, for the manufacturer, is whether to offer a threshold-based contracts or not. What if the manufacturer simply offered a constant-margin contract over all the periods? In this section we show that the manufacturer is better off offering a Δ-contract if her goal is to improve expected profits. In other words, we argue that in the space of linear threshold-based contracts, characterized by (p, Δ, K), the additional marginal payment, Δ is strictly positive in any optimal contract (i.e., $\Delta > 0$). Essentially, for any constant-margin contract, there exists a Δ-contract which improves the profit for the manufacturer.

For notational convenience, we define the constant margin contract as a P-contract and represent it by the contracting parameters $(p, 0, 0)$. Observe that, even though there is no threshold level K and no extra marginal payment, Δ, we can view this contrast as a Δ-contract with these two parameters set to zero. Our goal is to show that the manufacturer can improve her profit by offering the (p, Δ, K) contract with $\Delta > 0$ and $K > 0$. For any $p > 0$ and $\varepsilon \in (0, p]$ consider the following two contracts: $(p, 0, 0)$ and $(p - \varepsilon, \varepsilon, 0)$. Observe that the pay-off to the dealer is identical for all demand realizations for both contracts. Similarly, the pay-off to the manufacturer is identical for all demand realizations and hence both parties would be indifferent between these two contracts.

Next, fix $\varepsilon \in (0, p]$, and consider the contract $(p - \varepsilon, \varepsilon, K)$ where K is smaller than $g(e)$ where e solves $g'(e) = \beta/(p - \varepsilon)$. Notice that the dealer always exerts an effort e such that the marginal benefit from exerting the sales effort equates the marginal cost of exerting the effort. Thus, in this case the dealer always puts an high effort to exceed K and the total demand realization (including the impact of the dealer's effort) is identical to that observed under the contract $(p - \varepsilon, \varepsilon, 0)$. However, the manufacturer's payment to the dealer is reduced since the manufacturer only pays $p - \varepsilon$ for the first K units. Thus, the manufacturer is better off by offering $(p - \varepsilon, \varepsilon, K)$ instead of $(p - \varepsilon, \varepsilon, 0)$. This implies that the manufacturing firm is always better off (in terms of expected payoff) with a Δ-contract when $\Delta > 0$ and $K > 0$. A similar argument can be made for a D-contract (bonus payment). For a single period, Oyer (2000) shows that the D-contract is, in fact, an optimal contract.

An important question for the manufacturer is how to strike the right balance between higher expected profits and increased sales variance. To do so it is necessary to first characterize the sales HSP.

5. The Generalized Hockey Stick Phenomenon

In this section we study the sales HSP under both the contract forms, i.e., the Δ-contract and the D-contract. First, we characterize the dealer's optimal response when the incentive horizon consists of $T > 1$ time periods. We set up the dealer's optimization problem as a stochastic dynamic programming problem and show that the dealer's optimal effort levels form a submartingale. We also characterize the conditions when sales HSP is non-trivially observed.

5.1. The dealer's optimal response with a Δ-contract

As shown in Figure 1, consider the case when the manufacturer offers the dealer a (p, Δ, K) contract prior to the first time period. At the beginning of every time period the dealer must decide on the optimal effort level, e_t, such that his future expected profit, in time periods $t, \ldots, T$, is maximized. Recall the definition of D_t, the sales up to period t, from Eq. (3). We now define the *adjusted-threshold*, K_t as follows:

$$K_t = \max\{0, K - D_t\} = \max\{0, K_{t-1} - X_t - g(e_t)\}. \tag{6}$$

By definition $K_1 = K$. The recursive relationship in Eq. (6) follows from the definition of D_t.

First, observe that the dealer will at least exert a minimum effort, e_l^*, such that the marginal gain from exerting such an effort is equal to the marginal cost of exerting such an effort.

$$p\, g'(e_l^*) = \beta. \tag{7}$$

Furthermore, it is important to also note that, if in any period t, $D_t > K$ (i.e. $K_t \leq 0$), then for all time periods $t, \ldots, T$, the dealer exerts a constant effort, e_h^*, such that the marginal benefit equates the marginal cost, i.e.,

$$(p + \Delta) = \beta\, h(e_h^*) \tag{8}$$

Thus, once the total sales surpass the threshold K, the dealer's profit, from all future time periods, is a constant, $\mathcal{C}_t$, given by

$$\mathcal{C}_t = (p + \Delta)(T - t)\,(\mathbb{E}[X_t] + g(e_h^*)) - \beta(T - t)e_h^*. \tag{9}$$

Let $V_t(K_t)$ be the dealer's optimal expected profit from time periods $t, \ldots, T$ beginning in time period t. If $K_t \geq 0$, then $V_t(K_t)$, is given by the following:

$$\begin{aligned} V_t(K_t) = \max_{0 \leq e_t} & \int_0^{K_t - g(e_t)} \{p[X_t + g(e_t)] + V_{t+1}(K_t - X_t - g(e_t))\}\, f(X_t) dX_t \\ & + \int_{K_t - g(e_t)}^{\infty} \{(p + \Delta)[X_t + g(e_t)] - \Delta K_t + \mathcal{C}_t\}\, f(X_t) dX_t \end{aligned}$$

$$-\beta e_t \tag{10}$$

else, if $K_t < 0$, then $V_t(K_t) = C_t$. The other terminal condition is $V_{T+1}(K_{T+1}) = 0 \; \forall \; K_{T+1}$. The first term in the Eq. (10), corresponds to the profit when the sale in the current period, S_t, is below the adjusted threshold K_t. As a result of the effort exerted in the current time period, e_t, and the random demand signal observed thereafter, the adjusted threshold for the next time period, K_{t+1}, reduces to $K_t - X_t - g(e_t)$. Hence, the optimal expected profit from time period $t+1, \dots, T$ is given by $V_{t+1}(K_{t+1})$. The second term in (10) corresponds to the case when the sales, S_t, exceed the adjusted threshold K_t. In this case the total future period profits is constant $\mathcal{C}_t$, because the sales have already exceeded the incentive threshold, K, and hence the adjusted threshold $K_j \leq 0$ for all time periods, $j = t+1, \dots, T$ and the dealer exerts a constant effort e_h^*. It is easy to verify the concavity of the profit function in the last period, i.e. $t = T$, if the sales up to the last period, $D_T < K$. However, in general the expected profit function depends on the concavity of the impact-of-effort function, $g(\cdot)$, and the demand signal distribution, $F(\cdot)$. In the Appendix section, we derive some conditions, on the concavity of $g(\cdot)$, under which $V_t(K_t)$ is concave assuming the probability density function, $f(\cdot)$, is bounded above by a scalar $M < \infty$. The condition specifies that if $\frac{g''(e_t)}{g'(e_t)^2} \leq -\frac{\Delta M}{p}$ then $V_t(K_t)$ is a concave function.

Using Eq. (10) we characterize the dealer's optimal effort in Lemma 1. Let $\mathbf{1}_{\{a>b\}}$ denote the indicator function that is true when the condition $a > b$ is satisfied. Further, let $\mathbb{E}_{X_t,\dots,X_T}$ denote the expectation operator over all the random variables X_t through X_T.

Lemma 1. *Given a (p, Δ, K) contract the dealer's optimal effort, e_t^*, in any time period $t < T$, is characterized by the following optimality condition*

$$h(e_t^*) = \frac{1}{\beta}\left(p + \Delta \mathbb{E}_{X_t,\dots,X_T}\left[\mathbf{1}_{\left\{\sum_{j=t}^{T}\left(g(e_j^*)+X_j\right)>K_t\right\}}\right]\right). \tag{11}$$

Observe that, in Lemma 1, the left hand term is an increasing function (by Assumption 1) of effort e_t. Similarly, the right hand side is also an increasing function of e_t. It is possible to find at least one solution to Eq. (11). However, to allow for an unique effort level, and to simplify our analysis, we make the following assumption:

Assumption 2. *The solution to Eq. (11) is unique.*

A weaker assumption, instead of Assumption 2, would be to allow the dealer to choose only the lowest effort level in case of multiple solutions to Eq. (11).

Lemma 1 allows us to characterize the relationship between the dealer's optimal effort in any two consecutive periods. We state the results, which follow directly from Lemma 1, in Lemma 2.

Lemma 2. *Given a (p, Δ, K) contract the dealer's optimal efforts in any two consecutive time periods are related as follows:*

$$h(e_t^*) = \mathbb{E}[h(e_{t+1}^*)] \;\; \forall \;\; 1 \leq t < T. \tag{12}$$

Furthermore, the dealer's optimal effort is bounded as follows:

$$e_l^* \leq e_t^* \leq e_h^* \;\; \forall \;\; 1 \leq t \leq T. \tag{13}$$

A priori, given that the impact-of-effort function is concave, the HSP is not an obvious outcome. However, if Assumption 1 holds, then the dealer's expected sales effort in any time period is stochastically dominated by the expected sales effort in future time periods. We state the dominance relationship, between the dealer's expected optimal effort, in consecutive time periods, in Theorem 1.

Theorem 1. *If Assumption 1 holds then the dealer's optimal effort in time period t is stochastically dominated by the optimal effort in period $t+1$ for all $t = 1, \ldots, T-1$, i.e.,*

$$\mathbb{E}[e_{t+1}^*] \geq \mathbb{E}[e_t^*] \;\; \forall \; t = 1, \ldots, T-1. \tag{14}$$

A consequence of Theorem 1 is that the dealer's optimal efforts form a submartingale. If the demand signals, X_t $(t = 1, \ldots, T)$, are assumed to be iid then the expected sales, $\mathbb{E}[S_T] = \mathbb{E}[g(e_t^*) + X_t]$, in each time period also follow a similar stochastic dominance relationship resulting in the sales HSP. However, even if we do not assume that the demand signals are iid, in Theorem 2, we show that the sales HSP exists under certain conditions.

Theorem 2. *If Assumption 1 holds and the composite function $g \circ h^{-1}$ is concave the sales HSP exists. However, if $g \circ h^{-1}$ is convex then we observe a reversal in the sales HSP (see definition 2) even though the dealer's optimal efforts follow the stochastic dominance relationship shown in Eq.(15).*

5.2. General Demand Dependencies and Information Structure

The result in the above section can be generalized to the setting where the demand across time periods is not iid. To this end, consider a filtration $\mathcal{F} = \{\mathcal{F}_n : n = 1, 2, \ldots, T\}$ and assume that the demand process is adapted with respect to this filtration. Thus, at the end of period m the demand distribution for period n is the conditional distribution of D_n w.r.t. to $\mathcal{F}_m$. The above model captures the fact that the demand from period to period are dependent and also that the dealer can update his belief about the demand in future time periods as he gets closer to that time period. Let e_t represent the optimal effort of the dealer in time period t, then we can show the following result.

Theorem 3. *If Assumption 1 holds then the dealer's optimal effort process* $\{e_t : t = 1, \ldots, n\}$ *forms a submartingale with respect to the filtration* $\mathcal{F}$*, i.e.,*

$$\mathbb{E}[e^*_{t+1}|\mathcal{F}_t] \geq e^*_t \quad \forall \ t = 1, \ldots, T-1. \tag{15}$$

The above result can also be extended to get the same result as Theorem 2 for sales being a either a submartingale (exhibiting hockey stick) and a supermartingale (exhibiting a reverse hockey stick) depending on the composite function $g \circ h^{-1}$.

5.3. Sales Variance with a Δ-contract

A consequence of Theorem 2 is that in either case, i.e. with the sales HSP or the reverse sales HSP, the manufacturer experiences a variance in the orders placed by the dealer across multiple periods. In Theorem 4, we characterize the covariance of the dealer's optimal effort in time period $t+1$ with the demand signal in period.

Theorem 4. *The random demand signal in period* t*,* X_t*, is positively correlated to the random optimal effort,* e^*_{t+1}*, in period* $t+1$*, i.e.,* $cov(X_t, e_{t+1}) \geq 0$*.*

Consider a situation when the demand signals in every period are iid with variance σ^2. Theorem 4 implies that the variance of the total sales across all the periods, i.e, $\mathbb{V}[S]$ is larger than $T\sigma^2$. This variance in the total sales can hurt the manufacturer's profits. For example, inventory and production scheduling becomes challenging (Chen 2000) or transportation logistics costs could be higher. If these costs are high enough, the manufacturer may be better off not offering a threshold-based incentive. Earlier, in Section 4, we discussed why a manufacturer should offer a Δ-contract. However, if the cost associated with variance is large then it may be beneficial to switch to a constant-margin contract. In such a situation the manufacturer may lose on higher expected profits due to higher expected sales but gain on lower sales variance. The manufacturer may also choose to devise other strategies that enable her to reduce the impact of the sales HSP by adjusting the Δ-contract parameters appropriately. In Section 6, we discuss how additional information, and the use of correlated signals, can help in such situations.

5.4. Offering a Bonus Contract

In this section we study the other form of prevalent threshold-based contracts where the manufacturer offers the dealer a bonus payment, D, on reaching the threshold K, instead of an additional marginal payment of Δ. We denote the bonus contract by it parameters (p, D, K). We first characterize the dealer's optimal response when the incentive horizon consists of $T > 1$ time periods. Similar to the Δ-contract, we set up the dealer's optimization problem as a dynamic programming problem and show that the dealer's optimal effort levels form a submartingale. We also characterize the conditions when sales HSP is non-trivially observed.

5.5. The dealer's optimal response with a D-contract

As shown in Figure 1, consider the case when the manufacturer offers the dealer a (p, D, K) contract prior to the first time period. Similar to the Δ-contract we can express the dealer's optimal expected profit function as a DP formulation for the D-contract too. When $K_t \geq 0$, under a D-contract, the optimal expected profit function, $V_t(K_t)$ is as follows:

$$\begin{aligned} V_t(K_t) = \max_{0 \leq e_t} \; & p\mathbb{E}[X_t] + p\int_0^\infty g(e_t) f(X_t) dX_t + \int_0^{K_t - g(e_t)} V_{t+1}(K_t - X_t - g(e_t)) f(X_t) dX_t \\ & + \int_{K_t - g(e_t)}^\infty [D + \mathcal{C}_t] f(X_t) dX_t - \beta e_t \end{aligned} \tag{16}$$

where $\mathcal{C}_t$ is now defined as

$$\mathcal{C}_t = p(T-t)\left(\mathbb{E}[X_t] + g(e_l^*)\right) - \beta(T-t)e_l^*. \tag{17}$$

In Eq. (17) the effort level e_l^* is defined by Eq. (7). Furthermore, $V_t(K_t)$ takes a value of 0 when $K_t < 0$. Unlike the Δ-contract, the dealer's optimal effort now depends on the probability distribution of the random demand signal X_t (denoted by $f_{X_t}(\cdot)$). We characterize the dealer's optimal effort in Lemma 3. We omit the proof since it is very similar to the proof of Lemma 1.

LEMMA 3. *Given a (p, D, K) contract the dealer's optimal effort, e_t^*, in any time period $t < T$, is characterized by the following optimality condition:*

$$h(e_t^*) = \frac{1}{\beta}\left(p + D\mathbb{E}_{X_t,\ldots,X_T}\left[f_{X_t}\left(K_t - \sum_{j=t}^{T-1} X_j - \sum_{j=t}^{T} g(e_j^*)\right)\right]\right). \tag{18}$$

Using Lemma 3, and a Assumption 2, it is easy to verify that the dealer's optimal efforts form submartingale similar to the one discussed earlier in Theorem 1. So, Lemma 2 holds in this case too, and hence, the sales HSP is also observed with a D-contract.

5.6. Sales Variance with a D-contract

Unlike the Δ-contract case, we are not guaranteed a positive covariance between the optimal effort in a period and the random demand signal in the previous period. In some cases the covariance could be negative and hence the variance of sales, $\mathbb{V}[S]$, could be lower than than that of the underlying demand signal. We show an example when $T = 2$. Let the demand be uniform between 0 and a large level M. Here the first period effort is fixed and obtained by the above mentioned optimality equation. The second period effort then solves the following optimality equation:

$$h(e_2^*) = \frac{1}{\beta}[p + Df_U(K - X_1 - g(e_1^*) - g(e_2^*))],$$

where $f_U(\cdot)$ is the probability density function for the uniform distribution with support $[0, M]$. Thus, one can easily verify that there exists a threshold C such that if $X_1 > C$ then the optimal e_2^* is given by

$$h(e_2^*) = \frac{1}{\beta} p,$$

else it is given by

$$h(e_2^*) = \frac{1}{\beta}\left[p + \frac{1}{M}\right].$$

Observe that e_2^* is negatively correlated with X_1 and since $g(e_2^*)$ is increasing function of e_2^*, the overall variance of the sales might be reduced for the D-contract.

As a consequence the manufacturer may be better off by offering a D-contract, instead of Δ-contract, if sales variance across multiple periods is of concern.

6. Value of Better Information

As shown earlier, the manufacturer gains by offering a threshold-based incentive because expected sales are higher. However, the variance of sales can be expensive and reduce operational profits. The dealer's hidden information (demand signal), and his hidden actions (effort), result in this misalignment. One way of aligning incentives better (see Narayanan and Raman 2004) is to track additional business variables. In this section we show the value of such additional information in controlling the sales HSP. Specifically, we show that when the manufacturer observes additional information, correlated with the underlying demand signal, she is able to dampen the sales HSP by using such information to set the incentive threshold K. First, we demonstrate the elimination of the sales HSP when the observed signal is perfectly correlated with the underlying demand signal. Next, we extend the result to the partially correlated case. Essentially, the manufacturer is able to delay the computation of the threshold, K, and simultaneously link it to the demand signal uncertainty. Finally, we discuss an implementation plan for the partially correlated case. For most of the analysis in this section we consider the Δ-contract.

Consider the following scenario: Suppose Y_t $(t = 1, \ldots, T)$ are random iid variables (henceforth referred to as *market indicators*) and are verifiable (i.e., the manufacturer can write a contract where the payoffs depend on these signals), and positively correlated to the underlying market signals X_t. Examples of such market indicators include macro-economic indices observable to all entities in the economy. Consider a Δ-contract where the manufacturer can now modify the incentive scheme by linking the computation of the threshold, K, to the market indicator variables

$Y = \langle Y_1, \ldots, Y_T \rangle$. In this particular situation the computation of the actual value of the threshold is delayed until the last time period. For simplicity, we consider a threshold, K, computed as follows:

$$K \equiv \sum_{t=1}^{T} y_t + \bar{K}, \tag{19}$$

where y_t is the realization of the random variable Y_t and $\bar{K}$ is a non-negative constant. At the beginning of the first time period the manufacturer shares the threshold computation scheme (Eq. 19) with the dealer. The remaining sequence of events are exactly the same as before. Notice that, the dealer continues to observe his private market signals X_t and decides on his optimal effort based on the difference(error), $Z_t = X_t - Y_t$. It is worth noting that if there is a market indicator signal that is negatively correlated with the market demand signal then the manufacturer can use the negative of that market indicator signal.

Next, in Section 6.1, we discuss the effect of perfect correlation between Y and X.

6.1. Perfectly Observable Case

In this section we consider the case when the market indicators are perfectly correlated to the underlying demand signals. The dealer's optimization problem remains the same as before (Eq. 10).

First, in Proposition 1, we state the result that characterizes the dealer's optimal action (effort levels) under perfect correlation. Specifically, we show that the HSP is eliminated completely with perfect information. Recall our earlier definitions of the low effort level, e_l^*, and the high effort level, e_h^*, from equations (7) and (8) respectively.

PROPOSITION 1. *Let Y_t be perfectly correlated with X_t and $\mathbb{E}(Y_t) = \mathbb{E}(X_t)$, $\forall\ t = 1, \ldots, T$. Under a Δ-contract the dealer exerts the same effort in all time periods, i.e, the dealer either exerts a low effort, e_l^*, or a high effort e_h^*, in all time periods. Furthermore, there exists a $\widetilde{K} = T\, g(e_h^*)$ below which the dealer exerts a high effort level, e_h^*, and above which the dealer exerts a low effort level, e_l^*, in all time periods.*

Intuitively, it is easy to argue the result stated in Proposition 1. Since the impact-of-effort function, $g(\cdot)$, is concave and the cost-of-effort function is linear, it is optimal for the dealer to smoothen the total effort equally between all the periods. Proposition 1 shows that irrespective of the realizations of the underlying market signal, X_t, there exists a cutoff value $\widetilde{K}$ such that, if $\bar{K} > \widetilde{K}$, the optimal efforts satisfy $e_t^* = e_l^*\ \forall\ t = 1, \ldots, T$ else $e_t^* = e_h^*$. Thus, an important implication of Proposition 1 is that the hockey stick phenomenon can be completely eliminated using a perfectly correlated signal.

Since the impact-of-effort function, $g(\cdot)$, is an increasing function the following inequality relating the optimal efforts across the two periods must hold.

$$T\,g(e_l^*) \leq \sum_{t=1}^{T} g(e_t^*) \leq T\,g(e_h^*).$$

It is easy to see that the choice of $\bar{K}$ (which depends on the dealer's impact-of-effort function) controls the total sales in the perfectly correlated case. We summarize this result in Proposition 2 by showing how a manufacturer gains when the indicators are perfectly correlated. But first, we define path-wise dominance:

Definition 3. Given two random variables Z_1 and Z_2, we say Z_1 dominates Z_2 path-wise if $Z_1 \geq Z_2$ for all sample paths generated by all realizations of Z_1 and Z_2.

Proposition 2. 1. *If $\bar{K} > \widetilde{K}$ then the total sales across all the periods is higher path-wise when the threshold, K, is not based on a perfectly correlated indicator signal, Y.*

2. *However, if $\bar{K} < \widetilde{K}$ the total sales is larger path-wise when the threshold, K, is computed using a perfectly correlated market indicator Y.*

From Propositions 1 and 2 we observe the following: Setting a threshold, K, based on a perfectly correlated signal Y allows the manufacturer to gain by (i) eliminating the hockey stick phenomenon, and (ii) increasing total expected sales. However, if $\bar{K} > \widetilde{K}$, then the manufacturer faces a tradeoff between volume of sales and variability of sales. Specifically, if the manufacturer's objective is to maximize expected sales volume only, an arbitrary threshold, K, chosen a priori may suffice; on the other hand, if the manufacturer is concerned with the variability in the sales across the time periods, K should be based on a positively correlated signal Y. Thus, an important implication for the manufacturer is that, by choosing $\bar{K}$ appropriately, either the hockey stick effect can be completely eliminated or the total expected sales enhanced.

In reality though, choosing a market indicator signal that is perfectly correlated may be difficult. Next, we study the case when the market indicator signals are not perfectly correlated with the underlying demand signals.

6.2. Partially Observable Case

In this section we consider the case when the manufacturer (and the dealer) can observe a partially correlated market indicator signal and set the threshold, K, accordingly. However, it is challenging to derive a closed form analytical expressions for the HSP in a general setting. So, we demonstrate the effect of partial correlation by setting up an appropriate numerical experiment. For the numerical experiments we simulate a Δ-contract offered over two time periods, i.e., the case when $T = 2$.

We assume that $Z_t = (X_t - Y_t)$, where $t \in \{1,2\}$, is normally distributed with mean 50. We vary the standard deviations σ_t, in both periods, from 12 to 0 in steps of 3, i.e., $\sigma_t \in \{0,3,6,9,12\}$. The impact-of-effort function is defined as $g(e_t) \equiv 8\sqrt{e_t}$. The linear cost of exerting effort is defined as $v(e_t) = e_t$. The dealer's constant margin $p = 1$ and the additional marginal profit, Δ (on exceeding the threshold) is set to 0.8. The value of $\bar{K}$, in Eq. (19), is set to 10. Figure 2 summarizes the results

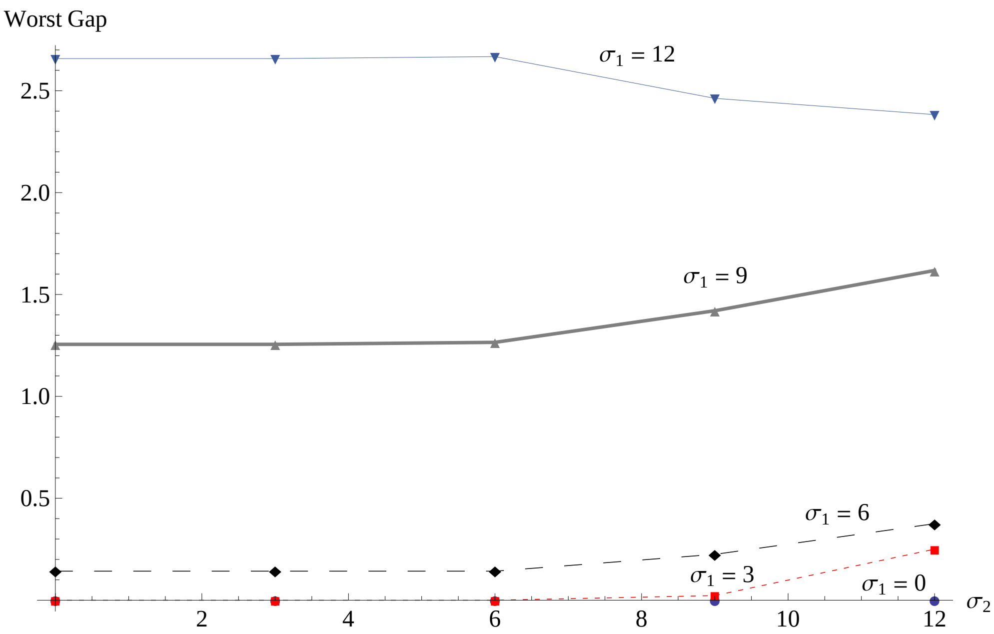

Figure 2 The worst case HSP gap when $T = 2$ for different σ_1 and σ_2 ($p = 1$, $\Delta = 0.8$, $\bar{K} = 10$, $g(e_t) = 8\sqrt{e_t}$).

of our numerical experiments. In Figure 2 we plot the maximum gap between the efforts across the two periods versus σ_2. Values are plotted for different levels of σ_1. As can be observed, for any level of σ_2, as σ_1 decreases the effort gap across the period diminishes implying that the HSP also diminishes. For the perfectly correlated case, i.e., when $\sigma_1 = 0$, the HSP vanishes. In general, as the variance of the error term, Z_t, increases the HSP is larger. These numerical experiments clearly demonstrate the benefits of setting the threshold based on correlated market indicator signals.

6.3. Implementation

In this section, we describe one way to implement a mechanism that allows the manufacturer to obtain information/signal that is very closely related to the underlying demand signal. We assume that the manufacturer works with M dealers and each dealer is in an independent geographical area. Such an assumption ensures that there is no competition effect, which we will study address later. Furthermore, for simplicity we assume that the threshold-based incentive is offered over two periods. We further assume, to keep our exposition simple, that the demand signals of all the dealers are positively correlated and the dealers are symmetric.

Without loss of generality we now consider the M^{th} dealer. In this case the manufacturer sets the incentive threshold, K, using the conditional expectation of the exogenous demand in the first period (i.e., the demand excluding the additional demand generated due to the effort of the dealer) plus a constant markup $\overline{K}$. Suppose S_1^j is the optimal sales of the j^{th} dealer in period 1 and X_1^j and e_1^{j*} are the corresponding demand signal and optimal effort exerted by the same dealer. Since $S_1^j = X_1^j + g(e_1^{j*})$ is monotone in X_1^j, the manufacturer can easily extract the exogenous demand signal X_1^j by observing the first period Sales S_1^j and using the optimality equations derived in Lemma 1. Note that here we are using the fact that the threshold value for a particular dealer is independent of his own effort and the analysis performed in Section 5 holds. Even if the sales function is not monotone in X_1^j, the manufacturer can use the sum of sales of all the other dealers instead of the sum of exogenous underlying demand signal. Thus, if all the M dealers are symmetric, the manufacturer sets a contract where the threshold for the M^{th} dealer is given by $\sum_{j=1}^{M-1} X_1^j/(M-1) + \overline{K}$. Using such a threshold value, and assumption that the demand signals are correlated, allows the manufacturer to postpone the computation of the incentive threshold and link it to the underlying signal variance. This allows the manufacturer to dampen the sales HSP and the smoothen the effort exerted by the dealer across the two time periods.

7. Dealer Competition

In this section we study how dealer competition affects the sales HSP. We consider a situation when two dealers, 1 and 2, compete in the market and exert effort (just as before) to increase sales. Both the dealers are exclusive to the manufacturer, i.e., they sell the manufacturer's product exclusively, and observe the same demand signal. To keep our analysis simple, we assume complete symmetry between the dealers. Essentially, we assume that the manufacturer offers the same Δ-contract to both the dealers and the demand split between the dealers is symmetric. Since we assume complete symmetry, it suffices to study the optimal response of dealer 1 only.

For notational ease, we identify the dealers by superscripts. So the effort exerted by dealer 1 in time period t is represented by e_t^1. Similarly, the effort exerted by dealer 2 is represented by e_t^2. In a competitive setting dealer 1's growth in sales is affected by the effort exerted by dealer 2 in the same time period. Thus, the total growth in sales for dealer 1, after exerting effort e_t^1 in time period t, depends on dealer 2's effort level e_t^2. We represent this impact-of-effort function by $\phi(e_t^1, e_t^2)$.

Again, for notational convenience, we represent the partial derivative of $\phi(e_t^1, e_t^2)$ with respect to e_t^1, and evaluated at $e_t^1 = e_t^2$, by $\phi_1(e_t^1, e_t^2)_{e_t^1 = e_t^2}$. Now, we define the function $\gamma(e_t^1)$ as follows:

$$\gamma(e_t^1) = \frac{1}{\phi_1(e_t^1, e_t^2)_{e_t^1 = e_t^2}} \tag{20}$$

Similar to the non-competitive setting, we make the following assumption:

ASSUMPTION 3. *Both,* $\phi(e_t^1, e_t^2)$*, and* $\gamma(e_t^1)$*, are continuous, increasing, and concave in* e_t^1*.*

In order to model the competitive aspects it is necessary to understand how competition affects the impact of sales effort. There are two possibilities: (1) competition intensifies, i.e., the competitor's effort reduces the impact of sales effort, or (2) competition is softened, i.e., the competitor's effort helps in increasing the impact of sales effort. The former situation may occur when the market size is limited (constant) and both dealers compete by stealing customers from their competition. However, if the market size is large enough, and a dealer can gain additional sales from the competitor's sales effort then competition is softened. For example, the competitor's advertisement campaigns may help in increasing the market size. To model these scenarios, and to allow comparison with the base non-competitive case, we define a scalar parameter ε such that

$$g'\left(e_t^1\right) = \gamma\left((1+\varepsilon)e_t^1\right) \tag{21}$$

While it is difficult to capture the true dynamics of competition, Eq. (21) allows us to capture the aforementioned scenarios very well. First, let us consider a non-competitive setting. Suppose dealer 1 exerts effort e_t^1 in time period t resulting in a sales growth of $g(e_t^1)$. Observe that, in Eq. (21), if $\varepsilon > 0$, then $\gamma(e_t^1) > g'(e_t^1)$. This implies that the dealer must exert lesser effort in the competitive case to have the same impact of effort as the non-competitive case. Thus, $\varepsilon > 0$ models the situation when competition is softened. However, if $\varepsilon < 0$, then the dealer must exert additional effort under competition to have the same impact as the non-competitive setting. Thus, $\varepsilon < 0$ models the scenario when competition intensifies.

First, in Lemma 4, we show that the dealer 1's optimal effort in each period follows the same optimality condition discussed in Lemma 1. Recall the definitions of K_t, from Eq. (6), and the indicator function $\mathbf{1}_{\{a>b\}}$.

LEMMA 4. *Suppose a manufacturer offers the same* Δ*-contract,* (Δ, K)*, to competing dealers 1 and 2. Dealer 1's optimal effort,* e_t^{1*}*, in any time period* $t < T$*, is characterized by the following optimality condition:*

$$\gamma(e_t^{1*}) = \frac{1}{\beta}\left(p + \Delta \mathbb{E}_{X_t,\ldots,X_T}\left[\mathbf{1}_{\sum_{j=t}^{T} \phi_1(e_t^{1*}, e_t^{2*})_{e_t^{1*} = e_t^{2*}} + X_j > K_t}\right]\right). \tag{22}$$

The proof for Lemma 4 follows the proof of Lemma 1. Hence, we omit the details of the proof. Under complete symmetry (when $e_t^{1*} = e_t^{2*}$) and Assumption 3, it is easy to verify that the HSP exists even under competition. Similar to the non-competitive case, we show the dominance of expected sales effort in Lemma 5 (again we omit the detailed proof since it follows from Jensen's inequaility).

LEMMA 5. *Given a (Δ, K) contract the dealer 1's optimal efforts in any two consecutive time periods are related as follows:*

$$\gamma(e_t^{1*}) \leq \gamma\left(\mathbb{E}(e_{t+1}^{1*})\right) \quad \forall \quad 1 \leq t < T. \tag{23}$$

An immediate consequence of Lemma 5 is that the sales HSP exists under competition too. But, an important question is whether the effect is severe or reduced? To understand this we focus our attention on a competitive situation with exactly two time periods, i.e., $T = 2$. Next, in Section 7.1, we study the effect of competition on the "worst-case" effort gap in a two time period model.

7.1. The worst case HSP gap when $T = 2$

In this section we consider the effect of competition on the worst case HSP gap in a two period model. Our goal is to compare the worst-case gap under competition with the non-competitive case discussed earlier in Section 5. As discussed in Section 3 we assume that the demand signals in each time period, $t = 1, 2$, are iid. First, for a given Δ-contract, we define the worst case HSP gap, $G(\Delta)$, as follows:

DEFINITION 4. For a given Δ-contract, the worst-case HSP gap is defined as the maximum gap between the optimal first period effort and the optimal expected second period effort, i.e.,

$$G(\Delta) = \max |\mathbb{E}[e_2^*] - e_1^*|. \tag{24}$$

In Figure 3 we illustrate the worst-case expected effort gap for the non-competitive case. From Lemma 2 we know that the effort levels in both the period are bounded between e_l^* and e_h^*. Since $h(\cdot)$ is assumed to be a concave function, the inverse function $h^{-1}(\cdot)$ must be a continuous convex function. The convex curve between p and $(p + \Delta)$ represents the expected second period effort $\mathbb{E}[e_2^*]$. Similarly, the straight line between p and $(p + \Delta)$ represents the optimal first period effort. The gap is maximum when the tangent to the convex curve is parallel to the straight line.

Now to characterize the maximum gap let us define a function $m(z) \equiv h^{-1}\left(\frac{z}{\beta}\right)$. Using the Mean Value Theorem it is easy to verify that the worst-case (maximum) gap, $G(\Delta)$, is

$$G(\Delta) = \int_0^z \left(\int_p^{p+\Delta} m'(z)\, dz \right) \Delta m'(y)\, dy \tag{25}$$

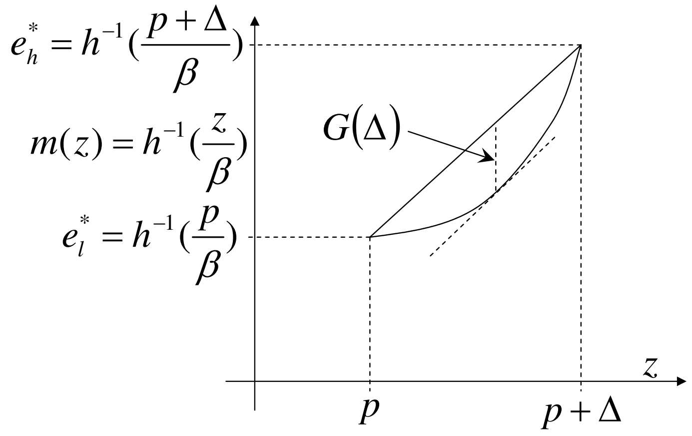

Figure 3 The worst case HSP gap when $T = 2$.

where, $m'(z)$, is the derivative with respect to z. Using Eq. (25) we show the relationship between the competitive and non-competitive worst-case gaps in Proposition 3. Let superscripts C and N represent the competitive and non-competitive scenarios for dealer 1 respectively.

Proposition 3. *Suppose the manufacturer offers the same Δ-contract to two competing dealers, and Assumption 3 holds, then the worst-case HSP gaps, for dealer 1, are related as follows:*

$$G^C(\Delta) = (1+\varepsilon)G^N(\Delta). \tag{26}$$

Proposition 3 shows that the worst-case HSP could decrease when the competition intensifies, i.e., $\varepsilon < 0$. However, when competition is softened, i.e., $\varepsilon > 0$, the worst-case gap increases suggesting the HSP could be exacerbated. But an important implication for the manufacturer is that she may be able to use dealer competition effectively to control the sales HSP.

8. Conclusions

When a firm offers an incentive scheme to its partner it typically does so to achieve certain objectives, e.g., increase expected profits or sales. However, sometimes such incentives result in undesirable, and unforseen, effects that could hurt the operational efficiency of the firm. Hence, it is necessary to design mechanisms that adjust the incentive parameters appropriately to eliminate such undesirable effects. The sales HSP is one such example that adversely affects operational costs of a manufacturing firm and is primarily a byproduct of a dealer's strategic behavior in exerting sales effort. In this paper we have shown that the sales HSP is non-trivially observed when a manufacturer offers a quota-based contract (either a Δ-contract or a D-contract) over multiple periods. The sales spike is a consequence of the pattern observed in the dealer's optimal sales effort choice over time. This results in higher sales variance for the manufacturer, especially with a Δ-contract. Under such circumstances the manufacturer faces a key tradeoff between higher profits due to higher expected sales versus larger costs due to higher variance in the orders placed by the dealer throughout the selling season.

One of the primary reasons for the sales HSP is the manufacturer's inability to choose the contracting parameters appropriately due to lack of underlying demand information. Eliciting such information, directly from the dealer, and monitoring contracting mechanisms that enforce such truthful revelation, is a challenging task. However, in some industries, it is possible to indirectly assess such information and use it in setting the incentive parameters. Specifically, the value of the threshold, K, plays an important role in controlling the sales HSP and sales variance. One way to address this issue is to base the choice of the threshold, K, on a positively correlated market indicator signal and delay its computation to as late as possible. The significant benefit of choosing such a threshold level, by linking it to the underlying market signal and delaying its computation, allows the manufacturer to simultaneously control the total expected sales and the variance of sales. This helps the manufacturer in coordinating the supply chain better and operate efficiently by making better production scheduling and inventory decisions.

Dealer competition also affects the sales HSP. With multiple competing dealers, a manufacturer has the ability to reduce the impact of the last period sales spike. The worst-case HSP decreases when competition intensifies. However, when competition is softened the worst-case gap increases suggesting the HSP could be exacerbated.

Acknowledgements

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9. Appendix: Proofs

Proof for Lemma 1.

The optimal profit function is given by

$$\begin{aligned}V_t(K_t) = \max_{0\leq e_t} &\int_0^{K_t-g(e_t)} \{p[X_t+g(e_t)] + V_{t+1}(K_t - X_t - g(e_t))\} f(X_t)dX_t \\ &+ \int_{K_t-g(e_t)}^{\infty} \{(p+\Delta)[X_t+g(e_t)] - \Delta K_t + \mathcal{C}_t\} f(X_t)dX_t \\ &-\beta e_t.\end{aligned}$$

In order the compute the optimal effort we will need to first compute the the partial derivative of $V_t(K_t)$ with respect to K_t, i.e., $\frac{\partial V_t(K_t)}{\partial K_t}$. To do so, we first look at the maximum achievable profit in the T^{th} period which is given by:

$$V_T(K_T) = \max_{0\leq e_T} \int_0^{K_T-g(e_T)} \{p[X_T+g(e_T)]\} f(X_T)dX_T$$

$$+\int_{K_T-g(e_T)}^{\infty}\{(p+\Delta)[X_T+g(e_T)]-\Delta K_T\}f(X_T)dX_T$$
$$-\beta e_T.$$

Observe that the optimal effort e_T^* is a function of K_T and $V_{T+1}(K_{T+1})\equiv 0$. First, we compute the last-period optimal effort, e_T^*, as follows. Using the first order optimality conditions it is easy to verify that

$$e_T^*=\begin{cases} e_T^* : \beta h(e_T^*)=[p+\Delta(1-F(K_T-g(e_T^*)))] \quad when \quad D_T<K \\ e_h^* : (p+\Delta)=\beta h(e_h^*) \quad when \quad D_T\geq K\end{cases} \tag{27}$$

Notice that, when $K_T\leq 0$, the partial derivative of $V_T(K_T)$ with respect to K_T, $\frac{\partial V_T(K_T)}{\partial K_T}$, is zero. However, when $K_T>0$ the we must use the Envelope Theorem to compute the derivative. Using the Envelope Theorem, it is easy to verify that:

$$\frac{\partial V_T(K_T)}{\partial K_T}=\begin{cases} -\Delta\mathbb{E}_{X_T}\left[\mathbf{1}_{\left\{g(e_T^*)+X_T>K_T\right\}}\right] & : K_t>0 \\ 0 & : K_T\leq 0\end{cases}. \tag{28}$$

We now make an assumption about the form of the partial derivative, $\frac{\partial V_t(K_t)}{\partial K_t}$, in any period t. Then, using the first order optimality condition, and an inductive argument, we will verify that our assumed form is correct. Let us suppose that

$$\frac{\partial V_t(K_t)}{\partial K_t}=\begin{cases} -\Delta\mathbb{E}_{X_t,\ldots,X_T}\left[\mathbf{1}_{\left\{\sum_{j=t}^T(g(e_j^*)+X_j)>K_t\right\}}\right] & : K_t>0 \\ 0 & : K_t\leq 0\end{cases}. \tag{29}$$

Eq. (29) will also lead us the desired optimality condition for the dealer's effort level in time period t. Let us assume the partial derivative form is valid for time period $t+1$, i.e.,

$$\frac{\partial V_{t+1}(K_{t+1})}{\partial K_{t+1}}=\begin{cases} -\Delta\mathbb{E}_{X_{t+1},\ldots,X_T}\left[\mathbf{1}_{\left\{\sum_{j=t+1}^T(g(e_j^*)+X_j)>K_{t+1}\right\}}\right] & : K_{t+1}>0 \\ 0 & : K_{t+1}\leq 0\end{cases}. \tag{30}$$

Using an inductive argument, we will show that the partial derivative holds for time period t too. Earlier, we have already shown that the partial derivative in the last period has a similar form. First we compute the partial derivative, with respect to effort e_t, which is given by $\frac{\partial V_t(K_t)}{\partial e_t}$

$$\begin{aligned}\frac{\partial V_t(K_t)}{\partial e_t} &= pg'(e_t)+\Delta g'(e_t)\mathbb{E}_{X_t}\left[\mathbf{1}_{\{g(e_t)+X_t>K_t\}}\right] \\ &\quad -g'(e_t)\int_0^{K_t-g(e_t)}\left.\frac{\partial V_{t+1}(K_{t+1})}{\partial K_{t+1}}\right|_{\left(K_{t+1}=K_t-g(e_t)-X_t\right)}f(X_t)dX_t \\ &\quad -\beta.\end{aligned}$$

Substituting for the value of $\left.\frac{\partial V_{t+1}(K_{t+1})}{\partial K_{t+1}}\right|_{\left(K_{t+1}=K_t-g(e_t)-X_t\right)}$ from Eq. (30) we get

$$\frac{\partial V_t(K_t)}{\partial e_t}=pg'(e_t)+\Delta g'(e_t)\mathbb{E}_{X_t}\left[\mathbf{1}_{\{g(e_t)+X_t>K_t\}}\right]$$

$$
\begin{aligned}
&+\Delta g'(e_t)\mathbb{E}_{X_t}\left[\frac{\partial V_{t+1}(K_{t+1})}{\partial K_{t+1}}\bigg|_{\left(K_{t+1}=K_t-g(e_t)-X_t\right)}\times \mathbf{1}_{\{(g(e_t)+X_t)\le K_t\}}\right]\\
&-\beta\\
=\;& pg'(e_t)+\Delta g'(e_t)\mathbb{E}_{X_t}\left[\mathbf{1}_{\{g(e_t)+X_t>K_t\}}\right]\\
&+\Delta g'(e_t)\mathbb{E}_{X_t,\dots,X_T}\left[\mathbf{1}_{\left\{g(e_t)+X_t+\sum_{j=t+1}^{T}(g(e_j^*)+X_j)>K_t\right\}}\times \mathbf{1}_{\{(g(e_t)+X_t)\le K_t\}}\right]\\
&-\beta\\
=\;& pg'(e_t)+\Delta g'(e_t)\mathbb{E}_{X_t}\left[\mathbf{1}_{\{g(e_t)+X_t>K_t\}}\right]\\
&+\Delta g'(e_t)\mathbb{E}_{X_t,\dots,X_T}\left[\mathbf{1}_{\left\{\sum_{j=t}^{T}(g(e_j^*)+X_j)>K_t\right\}}\right]\\
&-\beta
\end{aligned}
$$

Thus, the first order optimality condition implies that the first order condition must satisfy:

$$
p+\Delta\mathbb{E}_{X_t,\dots,X_T}\left[\mathbf{1}_{\left\{\sum_{j=t}^{T}(g(e_j^*)+X_j)>K_t\right\}}\right]=\beta h(e_t^*) \tag{31}
$$

However, we still need to verify that, if Eq. (31) holds, then $\frac{\partial V_t(K_t)}{\partial K_t}$ follows the same form mentioned earlier. Again, we use the Envelope Theorem to compute $\frac{\partial V_t(K_t)}{\partial K_t}$. It is easy to verify that, if $K_t>0$, then

$$
\begin{aligned}
\frac{\partial V_t(K_t)}{\partial K_t} &= -\Delta\mathbb{E}_{X_t,\dots,X_T}\left[\mathbf{1}_{\left\{g(e_t^*)+X_t+\sum_{j=t+1}^{T}(g(e_j^*)+X_j)>K_t\right\}}\times \mathbf{1}_{\{(g(e_t)+X_t)\le K_t\}}\right]\\
&= -\Delta\mathbb{E}_{X_t,\dots,X_T}\left[\mathbf{1}_{\left\{\sum_{j=t}^{T}(g(e_j^*)+X_j)>K_t\right\}}\right]
\end{aligned}
$$

However, when $K_t<0$, then $\frac{\partial V_t(K_t)}{\partial K_t}=0$. Thus, we have the proof by induction. ♦

Conditions for Concavity of the Dealer's Profit Under the Δ-Contract.

Suppose $f_J(\cdot)$ is the joint distribution of random variables $X_t,\dots,X_T$. Furthermore, we assume that $X_1,\dots,X_T$ are iid random variables. Now, the first partial derivative of the profit function with respect to e_t (in any time period t), $\dfrac{\partial V_t(K_t)}{\partial e_t}$, is

$$
\frac{\partial V_t(K_t)}{\partial e_t}=g'(e_t)\left[p+\Delta E_{X_t\dots X_T}1_{\sum_{j=t}^{T}(g(e_j)+X_j)>K_t}\right]-\beta.
$$

The second partial is given by

$$
\frac{\partial^2 V_t(K_t)}{\partial e_t^2}=g''(e_t)\,\mathrm{Q}+g'(e_t)\frac{\partial \mathrm{Q}}{\partial e_t}, \tag{32}
$$

where $\mathrm{Q}\equiv p+\Delta E_{X_t\dots X_T}\left[1_{\{\sum_{j=t}^{T}(g(e_j)+X_j)>K_t\}}\right]$. We know that

$$
p+\Delta\ge \mathrm{Q}\ge p. \tag{33}
$$

Let us define $Z \equiv \sum_{j=t}^{T} X_j$ and the convolution function of random variables $X_t, \ldots, X_T$ as $\phi(Z)$. Since $f_J(\cdot)$ is the joint density function of $X_t, \ldots, X_T$, we must have

$$\int_{\sum_{j=t}^{T}(g(e_j)+X_j)>K_t} f_J(X_t, \ldots, X_T)\, dX_t \ldots dX_T = 1 - \int_0^{K_t - \sum_{j=t}^{T} g(e_j)} \phi(Z)\, dZ \tag{34}$$

Using Eq. (34) we compute

$$\frac{\partial \mathrm{Q}}{\partial e_t} = g'(e_t)\phi\left(K_t - \sum_{j=t}^{T} g(e_j)\right) \tag{35}$$

Observe that, since the pdf, $f(\cdot)$, is bounded above by M, and the variables are iid we must have

$$\begin{aligned}\phi(Z) &= \int_{X_t=0}^{\infty} \cdots \int_{X_{T-1}=0}^{\infty} f(Z - \sum_{j=t}^{T-1} X_j) f(X_{T-1}) \ldots f(X_t)\, dX_{T-1} \ldots dX_t \\ &\leq M \int_{X_t=0}^{\infty} \cdots \int_{X_{T-1}=0}^{\infty} f(X_{T-1}) \ldots f(X_t)\, dX_{T-1} \ldots dX_t \\ & M.\end{aligned} \tag{36}$$

Thus, equations (36) and (35) imply

$$\tfrac{\partial \mathrm{Q}}{\partial e_t} \leq \Delta M\, g'(e_t) \tag{37}$$

To ensure concavity we must have $\dfrac{\partial^2 V_t(K_t)}{\partial e_t^2} \leq 0$. Using equation (37) this condition implies $\frac{g''(e_t)}{g'(e_t)^2} \leq -\frac{\Delta M}{\mathrm{Q}}$. Thus, from equation (33), $\dfrac{\partial^2 V_t(K_t)}{\partial e_t^2} \leq 0 \iff \dfrac{g''(e_t)}{g'(e_t)^2} \leq -\dfrac{\Delta M}{p}$. Hence, the profit function in any time period is concave if $\dfrac{g''(e_t)}{g'(e_t)^2} \leq -\dfrac{\Delta M}{p}$. ♦

Proof for Theorem 1.

Using Jensen's inequality it is easy to show that $h(e_t^*) \leq h\left(\mathbb{E}[e_{t+1}^*]\right)$. Since $h(\cdot)$ is an increasing function this implies that $e_t^* \leq \mathbb{E}[e_{t+1}^*]$. This further implies that $\mathbb{E}[e_t^*] \leq \mathbb{E}[e_{t+1}^*]$. ♦

Proof for Theorem 2.

Let us first consider the case when $g \circ h^{-1}(\cdot)$ is a concave function. From Eq. (11) we know that the optimal effort, e_t^*, satisfies the following condition:

$$e_t^* = h^{-1}\left(\frac{1}{\beta}\left(p + \Delta \mathbb{E}_{X_t,\ldots,X_T}\left[\mathbf{1}_{\left\{\sum_{j=t}^{T}\left(g(e_j^*)+X_j\right)>K_t\right\}}\right]\right)\right).$$

The expected sales in any period t, $\mathbb{E}[S_t]$, is given by $\mathbb{E}[X_t + g(e_t^*)]$. Therefore,

$$\begin{aligned}\mathbb{E}[S_t] &= \mathbb{E}[X_t] + \mathbb{E}[g(e_t^*)] \\ &= \mathbb{E}[X_t] + \mathbb{E}\left[g \circ h^{-1}\left(\frac{1}{\beta}\left(p + \Delta \mathbb{E}_{X_t,\ldots,X_T}\left[\mathbf{1}_{\left\{\sum_{j=t}^{T}\left(g(e_j^*)+X_j\right)>K_t\right\}}\right]\right)\right)\right].\end{aligned}$$

$$
\begin{aligned}
&\text{(by Jensen's inequality we have)}\\
&\leq \mathbb{E}[X_t] + g \circ h^{-1}\left(\mathbb{E}\left[\left(\frac{1}{\beta}\left(p + \Delta \mathbb{E}_{X_t,\ldots,X_T}\left[\mathbf{1}_{\left\{\sum_{j=t}^{T}\left(g(e_j^*)+X_j\right)>K_t\right\}}\right]\right)\right)\right]\right)\\
&= \mathbb{E}[X_t] + g \circ h^{-1}(h(e_{t+1}^*))\\
&= \mathbb{E}[X_t] + g(e_{t+1}^*)\\
&= \mathbb{E}[S_{t+1}]
\end{aligned}
$$

Thus, the sales HSP exists under this condition. However, following, similar arguments it can be shown that the inequality reserves when $g \circ h^{-1}(\cdot)$ is convex.

Proof for Theorem 4.

From Eq. (11) we know that the optimal effort, e_{t+1}^*, satisfies the following condition:

$$
e_{t+1}^* = h^{-1}\left(\frac{1}{\beta}\left(p + \Delta \mathbb{E}_{X_{t+1},\ldots,X_T}\left[\mathbf{1}_{\left\{\sum_{j=t+1}^{T}\left(g(e_j^*)+X_j\right)>K_{t+1}\right\}}\right]\right)\right). \tag{38}
$$

Observe that, in Eq. (38), as X_t increases, K_{t+1} decreases since $K_{t+1} = K - D_{t+1}$ (see definition of adjusted threshold in Eq. 6). Furthermore, $h^{-1}(\cdot)$ is convex and increasing (by Assumption 1). So, e_{t+1}^*, also increases as X_t increases. We now use a theorem proved in Schmidt (2003).

THEOREM 5 **(Schmidt)**. *Assume that J is convex and that X has a finite second moment. Then the inequality, $cov[\phi_1(X), \phi_2(X)] \geq 0$, holds for any two increasing functions $\phi_1, \phi_2 : J \to \mathbb{R}$ for which $\phi_1(X)$ and $\phi_2(X)$ have a finite second moment.*

Using, Theorem 5, it is easy to verify that $cov[e_{t+1}^*(X_t), h^{-1}(X_t)] \geq 0$. This further implies that e_{t+1}^* and X_t are positively correlated. ♦

Proof for Proposition 1.

With perfect correlation and perfect information, and the choice of the threshold $K = \sum_t^T x_t + \bar{K}$ (Eq. (19)), the dealer's profit function reduces to

$$
\Pi_\Delta = \begin{cases} p(K-\bar{K}) + p\sum_{t=1}^{T} g(e_t) - \beta \sum_{t=1}^{T} e_t & : \sum_{t=1}^{T} g(e_t) < \bar{K} \\ (p+\Delta)(K-\bar{K}) + (p+\Delta)\sum_{t=1}^{T} g(e_t) - \Delta K - \beta \sum_{t=1}^{T} e_t & : \sum_{t=1}^{T} g(e_t) \geq \bar{K} \end{cases} \tag{39}
$$

It is easy to verify the dealer's profit function is a deterministic concave function of the vector of efforts, i.e., $(e_1, \ldots, e_t, \ldots, e_T)$. To maximize his profit the dealer must solve the following two optimization problems:

$$
\begin{aligned}
\max_{e_1,\ldots,e_T} \; & p\sum_{t=1}^{T} g(e_t) - \beta \sum_{t=1}^{T} e_t \\
s.t. \quad & e_1, \ldots, e_T \geq 0
\end{aligned} \tag{40}
$$

$$
\max_{e_1,\ldots,e_T} \; (p+\Delta)\sum_{t=1}^{T} g(e_t) - \beta \sum_{t=1}^{T} e_t \tag{41}
$$

$$s.t. \qquad e_1,\ldots,e_T \geq 0$$

and choose the effort levels that result in the maximum profit. Since, both Eq. (40) and Eq. (41), are concave functions, the the optimal effort in each time period is either e_l^* (optimal solution to Eq. 40) or e_h^* (optimal solution to Eq. 41), where e_l^* is defined by Eq. (7) and e_h^* is defined by Eq. (8). Hence, the dealer exerts the same effort in all time periods. Thus, the sales HSP is eliminated under perfect information. Essentially, the steps followed by the dealer are the following: (1) Compute e_l^* and e_h^*, (2) Compare the which if $\sum_{t=1}^{T} e_l^* < \bar{K}$ and $\sum_{t=1}^{T} e_h^* \geq \bar{K}$ choose the high effort, e_h^* in each period, else (4) $\sum_{t=1}^{T} e_h^* < \bar{K}$ choose the low effort, e_l^* in each period.

Now, consider the case when $\bar{K} > T\,g(e_h^*)$. Under this condition $\sum_{t=1}^{T} e_h^* \geq \bar{K}$ is never satisfied and the dealer must choose the lower effort in every time period. If the condition doesn't hold then the dealer will always choose the higher effort level, e_h^*, in each period. ♦

Proof for Proposition 2.

Earlier in Proposition 1 we proved that the dealer either exerts a constant low effort, e_l^*, in all time periods or a high effort, e_h^*, in all time periods. This depends on the value of $\bar{K}$ chosen. We saw that if $\bar{K} > \widetilde{K} = Tg(e_h^*)$, then the dealer always chooses the lower effort for every realization of X_t, $t = 1,\ldots,T$. Hence, had the manufacturer chosen any arbitrary threshold, K, (not based on a perfectly correlated signal) the dealer's effort in any time period would be at least as large as e_l^* (see Lemma 2). However, if the manufacturer chose $\bar{K} < \widetilde{K} = Tg(e_h^*)$, the dealer exerts the highest effort in all time periods and hence the sales are the highest of any realization of the demand signals. This is not the case when K is chosen arbitrarily (not based on a correlated signal), since (see Lemma 2) the dealer's effort is strictly bounded as follows: $e_l^* \leq e_t^* \leq e_h^*$. Thus, path-wise dominance is obtained based on the choice of $\bar{K}$ under perfect correlation and information.

Proof for Proposition 3.

Recall that the worst-case gap, $G(\Delta)$, is defined by Eq. (25).

$$G(\Delta) = sup_{z\in[p,p+\Delta]} \int_p^z \left(\int_p^{p+\Delta} \frac{m'(z)}{\Delta}\, dz \right) - m'(y)\, dy$$

Further, defining $g_\epsilon(e) \equiv \gamma((1+\epsilon)e)$, we have

$$h_\epsilon(e) = \frac{1}{\gamma((1+\epsilon)e)} \text{ and } m_\varepsilon(\cdot) \equiv h_\varepsilon^{-1}(\cdot).$$

Now, consider any two functions, $f_1(\cdot)$ and $f_2(\cdot)$, such that $f_2(y) = f_1((1+\epsilon)y)$. Then it is easy to verify that $f_1^{-1}(y) = (1+\epsilon)f_2^{-1}(y)$. This implies that

$$(1+\epsilon)m_\epsilon(z) = m_0(z),$$

which in turn implies $(1+\epsilon)m'_\epsilon(z) = m'_0(z)$. Thus, using the definition of the worst case gap, we get the desired result. This completes the proof.

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