COMMENTARY

Mathematical finance

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Till the late sixties a banker's life was the source of envy for most of us: 3–6–3 formula was all that was needed to explain it – borrow at 3%, lend at 6% and play golf at 3 p.m. Over the last 50 years this has changed in a big way – no one can dream of having a quintessential banker's life any more. What were the changes that caused this dream-like life-style to evaporate? In what way are these changes affecting us? What are the common threads that are emerging and finally where are they likely to lead us? These are some of the points to ponder in this commentary on mathematical finance.

What is finance all about?

One may say finance is in vogue since the beginning of the world, in some form or the other and so we all know enough about it. However, while learning finance as a subject in classroom, in a 'corporate' context we are told that 'finance' is about answering three fundamental questions: One, where to invest the money? Two, how to get the money? Three, how to keep the corporate going? In management jargon, finance is about capital investment/capital budgeting decision; choice of source(s) of funding – debt/equity mix and finally, working capital decisions.

How to make these decisions?

To make these decisions consistently, one needs to recognize that the answers to these questions are linked to each other – like the two adjoining sides of a triangle. But one concept which is connected to all the three corners of the finance-triangle is called 'value'. However, this is a static view. The dynamic view requires that we include 'time' as well. Thus, the two essential ingredients of finance as a subject or profession are: concept of 'value' of an asset/liability and the changes in it, primarily with respect to 'time'. Typically, the metric or the measure of value, in the real world, is money and time is measured in clock-time or chronological time. So, in some sense, finance is about the change in value over time.

What is mathematical finance?

The 'value' is very often measured as the net present worth of (expected) future cash flows discounted at some rate. Commonly, it can be put in a formula as:

$$\mathrm{NPV} = -I_0 + \sum_{i=1}^{i=n} \frac{C_i}{(1+r_i)^i},$$

where NPV stands for net present value, $-I_0$ denotes initial investment, C_i (expected) future cash flow in period i, r_i the discount rate for period i and n is the number of periods considered for a given asset (or a liability).

As soon as we realize that in the real world initial investment, expected future cash flow, interest rate and the number of periods are not deterministic but uncertain, we recognize that the world has a lot of risk. In a formal sense the world is probabilistic or stochastic and our attempt is to get 'best estimates', 'expected outcomes', 'most likely scenario', etc.

While the body of knowledge on risk is ever increasing, the early formal work on risk was covered under actuarial mathematics. We all know that historically economics was studied as a branch of moral philosophy or moral science. In the 20th century it became more of a science. Today, some experts are of the view that 'mathematical finance' closely resembles 'applied microeconomic theory'. Expectedly, there are many variants under mathematical finance. The label given to a variant primarily depends on the focus/approach taken for problem formulation and/or solution. One comes across various labels, such as quantitative finance, computational finance, statistical finance, financial econometrics, analytical finance, empirical finance, etc.

Tools of the trade

Typically, on one end of the spectrum, a common person tends to believe that one needs to learn or needs to be a priori a 'rocket scientist' to enter mathematical finance; whereas at the other end of the spectrum mathematicians believe that entry-level requirements are rather modest because one needs to know some calculus, statistics, probability and linear algebra, and combine them with finance through concepts of stock and bond prices to arrive at Markowitz portfolio theory, capital asset pricing model, Black–Scholes–Merton option pricing formula, etc. However, a simplified view is that NPV is calculated using mathematical models of variables in the denominator and those in the numerator – separately or simultaneously. The variable in the numerator – cash flow – can be modelled in different ways with the help of stochastic processes. Similarly, the variable in the denominator – discount rate or interest rate – could be modelled, once again, with the help of stochastic processes along with the life of the asset as a random variable. When closed-form solutions are not available we use numerical methods and/or Monte Carlo simulations to arrive at the estimates of value. Given the computing power available at the laptop level – with multi-processors and several GB of RAM – many such problems could be solved while one is on the move; which also means that we are capable of building bigger models.

Move away from 3–6–3 formula

Perhaps, one of the important contributions came from Kiyosi Ito in the 1940s when he proposed a Taylor series expansion-based method of solving stochastic differential equations. This approach, now called the Ito–Doeblin lemma, set the stage for development of probabilistic approach to finance.

Another set of mathematical tools which had their roots in the Second

World War as (military) operations and research (OR) when applied to finance-related problems to find the 'best' or 'optimal' combinations of various assets, thanks to Harry Markowitz, brought in the portfolio theory. Formal application of these concepts is often found in the mutual funds industry.

Many believe that the tectonic shift which took place in the early seventies was responsible for beginning of the end of the 3–6–3 era of banking – those who refused to change perished like dinosaurs, cocooned in the regulatory web (a mixed-metaphor indeed!). It is not as if something earth-shattering happened in the seventies – there were people who were specializing in speculating on the outcome of certain events and taking bets on them, for ages. As these contingent claims or bets were primarily between private parties, there was a lack of transparency and often a vast information asymmetry existed; therefore, ascertaining and knowing the fairness of the bets and enforcement of the large bets or 'contracts' was always problematic. Actually, an open problem at that time was to have a consistent assessment of the 'fair' value of such bets based on publicly available information.

The first major change came with the option-pricing formula of Black–Scholes–Merton which was the main point of departure from the traditional finance theory and 'mathematization' of finance, in some sense, began. This formula, based on publicly available information, was able to price an options contract (to exercise or not to exercise) consistently. The assumptions about the underlying stochastic process and closed-form solution were other important points. They also established that under certain conditions (no-arbitrage) continuous hedging of the bets was possible. On the back of this success, slowly and gradually, standardized contracts were introduced on various indexes, stocks, bonds and commodities. The (stock and commodity) Exchanges in the United States actively took part in organizing the trading of these standardized contracts, thus reducing the probability of non-performance by either party. In other words, the credit-risk came down and with that the associated repetitive costs of enforcement, monitoring, etc. at individual level were also reduced. By the way, this role of Exchanges and many more things put together is formally called market microstructure.

The second major change that took place was at the end of the Cold War – it affected the lives of mathematicians and scientists working in the (erstwhile) Eastern-bloc countries. A large amount of intellectual resources were freed and Wall Street became the favoured destination to put theory into practice, thus fuelling the growth of newer varieties of contracts – by now being called non-standard or exotic 'derivatives'. More and more such contracts were introduced on previously unthinkable 'variables', like weather, inflation, volatility, etc. to name a few.

The third major change has obviously come with cost-efficient availability of computing and programing power and ease of access through communication networks, primarily internet. With each generation of computers, mathematical models became more sophisticated and given internet-based connectivity the hitherto unthinkable happened – we have a global market place buzzing with activity 24×7.

In the late eighties came computer-savvy investment managers with a large amount of money at their disposal on behalf of the super-rich – called high net worth individuals (HNIs). Larger and larger bets were taken and the players had gathered enough fire-power to move the market in the desired direction. These investment vehicles are sometimes referred to as hedge funds. Between these set of investors newer exotic private financial contracts were written and the risk arising out of these over-the-counter (OTC) contracts was hedged by means of the exchange-traded plain vanilla options and future contracts. Further, given the computing power at their disposal hedge funds were among the pioneers of what is popularly known as algorithmic trading.

While all this was bright and sunny, the darker clouds were not far behind. The untamed beast of human greed – beyond mathematical modelling – reared its ugly head. Unlimited risk-taking either in connivance of authorities or by keeping them in the dark led to the collapse of many long-standing and successful business houses and banks: 'greed is good' was the mantra.

Regulators and governments all over the world – to put it mildly – were taken by surprise by these developments. To some extent, they were not ready for these evolutionary changes. As a reaction to the changes taking place in the market structure they started monitoring the markets more closely, introduced many rules and regulations to protect the individual investors and market institutions from themselves; albeit with limited success.

Many experts believe that the fall in the markets in October 2008 – often called the sub-prime crisis – was created because various assets with different risks were bundled (put in a tranche) and sold to unsuspecting investors. This brought out issues relating to 'volatility forecasting', 'operational risk', 'capital adequacy', 'systemic stability', etc. These events also invited a lot of criticism of the innovations in the field of finance, and people publicly debated banning financial innovations. And the trend continues….

What next?

While mathematical models have helped us in coming up with 'innovative financial instruments' at the micro-end of the activities, there is an emerging body of knowledge at the macro-end as well where attempts are being made to model the markets. Interesting outcomes are expected from these models with introduction of behaviour at the individual level and its macro manifestation, sometimes called bubbles. Research in 'behavioural finance' is gaining momentum.

Among the market participants, the emerging trend is to have better and faster communication links along with locating market participants' computer systems in physical proximity of Exchanges – called colocation – to get a shorter latency period and to have the prices as 'fresh' as possible. In effect, this is an era of amalgamation of algorithmic trading and ultra-high frequency trading.

Expectedly, the interventions by various governments to 'bail-out' financial institutions are not well received by many of the small and individual investors. They feel that their hard-earned money collected through taxes is being doled out to guarantee fat bonuses to already wealthy bankers. As such, 'Occupy Wall Street' kinds of mass movements with political undertones of class struggle are likely to gain momentum in the short run.

The researchers in mathematical finance, obviously, either belong to or are embedded in the same socio-cultural-economic milieu as the rest of the society. Therefore, they cannot be oblivious of the changes taking place in the society at large. They need to put efforts to understand the human behaviour more closely and make models more realistic, in the days to come. Perhaps, by the time the next 'Wall Street' movie comes up some of these things may have become a reality.

Further exploration

[Disclaimer: This list is neither the best nor the most exhaustive on the topics. It is more like a list of some movies/books/websites which may be of interest to some of the readers.]

Some movies

- Boiler Room (2000)
- Rogue Trader (1999)
- Wall Street (1987)
- Wall Street: Money Never Sleeps (2010)

Some books

(i) Popular reading

Szpiro, G. G., *Pricing the Future: Finance, Physics, and the 300-Year Journey to the Black–Scholes Equation*, Basic Books, 2012.
Derman, E., *My Life as a Quant: Reflections on Physics and Finance*, Wiley, 2004.
Das, S., *Traders, Guns and Money: Knowns and Unknowns in the Dazzling World of Derivatives*, Pearson, 2006.
Bernstein, P. L., *Against the Gods: The Remarkable Story of Risk*, John Wiley and Sons, 1996.
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Lewis, M., *Liar's Poker: Rising Through the Wreckage on Wall Street*, Norton, 1989.
Malkiel, B. G., *A Random Walk Down Wall Street*, Norton, 1987.
Macay, C., 1841 (original publication) *Extraordinary Popular Delusions and the Madness of Crowds*, Templeton Foundation Press, 1985.

(ii) Technical reading

Capinski, M. and Zastawniak, T., *Mathematics for Finance: An Introduction to Financial Engineering*, Springer, 2003.
Wilmott, P., *Derivatives: The Theory and Practice of Financial Engineering*, John Wiley, 1998.
Karatzas, I. and Shreve, S. E., *Brownian Motion and Stochastic Calculus*, Springer-Verlag, 1991.
Karatzas, I. and Shreve, S. E., *Methods of Mathematical Finance*, Springer-Verlag, 1998.
Musiela, M. and Rutkowski, M., *Martingale Methods in Financial Modelling*, Springer-Verlag, 1997.
Barucci, E., *Financial Markets Theory: Equilibrium, Efficiency and Information*, Springer-Verlag, 2003.
O'Hara, M., *Market Microstructure Theory*, Blackwell, 1995.
Pliska, S. R., *Introduction to Mathematical Finance: Discrete Time Models*, Blackwell Publishers, 1997.
Duffie, D., *Dynamic Asset Pricing Models*, Princeton University Press, 2001, 3rd edn.
Kallianpur, G. and Karandikar, R. L., *Introduction to Option Pricing Theory*, Birkhauser, 1999.
Hamilton, J. D., *Time Series Analysis*, Princeton University Press, 1994.

Some websites

Market regulators – International Organization of Securities Commissions (IOSCO); www.iosco.org
Financial stability – Bank for International Settlements (BIS); www.bis.org
Market institutions – World Federation of Exchanges (WFE); www.world-exchanges.org
Hedge funds – Alternative Investments Management Association (AIMA); www.aima.org

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