The Importance of Decoupling Recurrent and Disruption Risks in a Supply Chain.

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ABSTRACT

This paper focuses on the importance of decoupling recurrent supply risk and disruption risk when planning appropriate mitigation strategies. We show that bundling the two uncertainties leads a manager to underutilize a reliable source while over utilizing a cheaper but less reliable supplier. As in Dada, Petruzzi and Schwarz [6], we show that increasing quantity from a cheaper but less reliable source is an effective risk mitigation strategy if most of the supply risk growth comes from an increase in recurrent uncertainty. In contrast, we show that a firm should order more from a reliable source and less from a cheaper but less reliable source if most of the supply risk growth comes from an increase in disruption probability.

1. INTRODUCTION

Chopra and Sodhi [5] discuss several supply risks that a manager must account for when planning suitable mitigation strategies. In this paper we focus on two of the risks categorized by them - disruptions and delays. Delays can be viewed as recurrent risks, whereas disruptions correspond to the interruption of supply. Our goal is to highlight the importance of recognizing the two risks as being distinct. We show that bundling the two risks can lead to an over utilization of cheaper suppliers and an under utilization of reliable suppliers. We also show that the mitigation strategies adopted are different depending upon whether most of the supply risk is recurrent or results from disruption.

A classic example of disruption is the shortage of flu vaccine in Fall 2004 that occurred in the United States after 46 million doses produced by Chiron, one of only two suppliers, were condemned because of bacterial contamination [12]. This shortage led to rationing in most states and severe price gouging in some cases. The lack of a reliable backup source of supply severely affected the nation's vaccine supply. In contrast, Canada had no such problem. In spite of a much smaller population base, Canada relies on more suppliers which make it less vulnerable to disruption from any one supplier. Another example is the March 2000 fire at the Philips microchip plant in Albuquerque, N.M. [19]. That plant supplied chips to both Nokia and Ericsson. Nokia learned of the impending chip shortage in just three days and took advantage of their multi-tiered supplier strategy to obtain chips from other sources. Ericsson, however, could not avoid a production shutdown because it was sourcing only from that plant. As a result, the company suffered $400 million in lost sales.

In both examples, one party benefited from mitigating disruption risk by having additional suppliers. In this paper, we offer a possible explanation for the different actions taken by the two parties. We show that bundling of disruption and recurrent risk results in situations where the reliable supplier is not used when it should have been. In general, bundling disruption and recurrent supply uncertainty results in an over utilization of the cheaper supplier and an under utilization of the reliable supplier.

We also show that the source of supply risk affects the relative use of cheaper suppliers and more reliable suppliers. Similar to the conclusions of Dada, Petruzzi, and Schwarz [6], we show that increased ordering from cheaper suppliers is an effective mitigation strategy if an increase in supply risk results from an increase in recurrent supply uncertainty. In contrast, we show that increased use of the reliable supplier and decreased use of the cheaper but less reliable supplier is a better mitigation strategy if an increase in supply uncertainty results from an increase in disruption risk.

Although our results are derived in a single period setting, we illustrate the difference between bundling and decoupling of recurrent and disruption risks by considering the supply received by a manager placing and receiving orders over twenty periods as shown in Table 1. The manager orders 100 units each period and receives supply as shown in the first column. We model recurrent supply uncertainty by assuming that the delivered quantity is subject to variability and that the lead time is fixed.

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Insert Table 1 About Here. Proposed caption:

Table 1: Delivery Log

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If the manager views the fluctuation in supply quantity as coming from a single source, she will use the entire column of supply quantities to estimate uncertainty. Using the supply data in the first column she estimates supply uncertainty to be represented by an average delivery of 86 units with a standard deviation of 38.60 when orders for 100 units are placed. In this case, the manager has bundled all uncertainty. A closer look at the data reveals a few days with zero supply. If we interpret zero supply to be a disruption and all other fluctuation to be recurrent supply uncertainty, the manager should interpret supply uncertainty differently. Considering the same data in the "Sorted by size" column reveals that disruption occurs in 3 of 20 instances and supply quantity fluctuates for other reasons in 17 of 20 instances. Thus, the manager should estimate supply uncertainty in two parts - a disruption probability of 15 percent and, in case of no disruption, a supply distribution with a mean of 101 units with a standard deviation of 11.87 units (when orders for 100 units are placed). In this case the supply manager correctly decouples disruption and recurrent supply uncertainty.

There has been a good amount of conceptual work regarding supply chain risks in general, and disruption uncertainty in particular. Mitroff and Alpasan [10] provide strategic tools to help identify stress causes and their impact on a firm's preparedness towards disruptive events. Chapman et al [3] discuss supply chain vulnerabilities by enumerating sources of disruptions and analyzing the impacts of each. Zsidisin et al [23] observe how seven supply chain champions measure and manage risk sources. At a more technical level, Qi [16] provides centralized and decentralized coordination models and tests a firm's operating plan in a one-supplier one-retailer setting in the presence of disruption risk. Kleindorfer and Saad [9] chart a conceptual framework that trades off risk mitigating investments against potential losses caused by supply disruption. Gaonkar and Viswanadham [7] also build an empirical framework that addresses the question of choosing a set of suppliers that minimizes loss caused by deviation, disruption, and disaster risks.

Christopher and Lee [4] draw upon additional disruption instances and also illustrate that lack of confidence and panic lead stakeholders to make irrational supply chain decisions. Sheffi [18] revisits various supply chain risk reduction mechanisms (visibility, multiple sourcing, collaboration, pooling, and postponement) and addresses the critical issue of how a firm should apply them in the presence of a terrorism threat, while maintaining operational effectiveness.

Although our work can be related to the work on random yields as in Yano and Lee [22], the value of decoupling recurrent from disruption risks is an issue that has not been considered in the random yields literature. There has been recent work that focuses on deriving optimal multi-period ordering policies where it is assumed that the current state of the supply process is known (either 'available' or 'not available'). This includes Weiss and Rosenthal [20] who integrate disruption uncertainty in EOQ inventory systems by developing optimal inventory policies in anticipation of a random length interruption in the supply or demand process, but where the interruption starting time is known in advance. Parlar [13] and Parlar and Perry [14] invoke renewal theory to model how the multi-period (q, r) replenishment policies can be extended to a setting that includes supply interruptions of random lengths of time. They derive average cost and reordering policies for when the supplier is available and not available, assuming that the distributions of the amount of time for both instances are known.

The fact that dual sourcing improves performance is demonstrated in several settings, including when there is no supply uncertainty (Bulinskaya [2], Whittmore and Saunders [21], Moinzadeh and Nahmias [11]) and when there is supply or demand uncertainty (Anupindi and Akella [1], Gerchak and Parlar [8], Parlar and Wang [15], Ramasesh et al. [17]). In contrast to the above literature, which focuses on how best to use multiple sources, we focus on how bundling of uncertainties affects a manager's use of reliable backup suppliers.

Our paper is closely linked to the work of Dada et al. [6]. They consider the problem of a newsvendor supplied by multiple suppliers with varying cost and reliability. They study properties of the optimal solution and show that cost generally takes priority over reliability when selecting suppliers. While we briefly discuss the selection of suppliers, our paper is much more focused on the relative use of the cheaper supplier and the reliable supplier once both have been selected. Our model expands on the insights of Dada et al. [6] by separately considering whether the supply risk is primarily recurrent or because of disruption. We show that increased use of the cheaper supplier is optimal if the growth in supply uncertainty is primarily from an increase in recurrent supply uncertainty. In contrast, we show that reliability takes priority over cost and it is optimal to increase the use of the reliable supplier and decrease the use of the cheaper supplier if most of the growth in supply uncertainty results from disruption.

2. ERRORS FROM BUNDLING WITH TWO SUPPLIERS: ONE PRONE TO DISRUPTION, ONE PERFECTLY RELIABLE.

Consider a single period problem where the buyer faces a fixed demand D over the coming period. The buyer has two supply options - one cheaper, but prone to disruption and recurrent supply risk (referred to as the first supplier) and the other perfectly reliable and responsive, but more expensive (referred to as the reliable supplier). The first supplier may have supply disrupted with probability p, in which case the buyer receives a supply of 0. If there is no disruption (with probability 1-p), the amount delivered is a symmetric random variable, X, with density function $f(X)$ having a mean of S (the quantity ordered) and standard deviation σ_X. Note that in our model, supply may exceed the order quantity. Such a situation may arise in a context where yields are random (such as the flu vaccine or semi-conductors) and the contracts are on production starts. We also note that this assumption simplifies the analysis and allows us to draw useful managerial insights. Each unsold unit at the end of the period is charged an overage cost of C_o and each unit of unmet demand is charged a shortage cost of C_u. We restrict attention to the case where $C_u > C_o$.

The reliable supplier has no disruption or recurrent supply uncertainty, i.e., the supplier is able to deliver exactly the quantity ordered. Responsiveness of the reliable supplier allows the manager to place her order after observing the response of the first supplier and yet receive supply in time to meet demand. This reliability and responsiveness, however, comes at a price. The reliable supplier charges a premium and requires the manager to reserve I units (at a unit cost of \$$h$ per unit) at the beginning of the period before knowing the outcome of supply from the first supplier. Once the outcome from the first supplier is known the manager can then order any quantity up to the I units reserved at an exercise price of \$$e$ per unit. If $e+h \geq C_u$, the manager does not use the reliable supplier because under stocking costs less than getting product from the reliable supplier. Thus, we assume that $e+h < C_u$. If $h \geq C_o$, the manager does not reserve any capacity from the reliable supplier in the absence of disruption, preferring to over order from the cheaper supplier. Thus, we assume that $h < C_o$. Also, it is reasonable to assume that the total cost from the reliable supplier $e+h$ exceeds the cost of overstocking C_o of purchases from the cheaper supplier, i.e., $e+h > C_o$. The manager's goal is to minimize total expected costs.

The sequence of events is as follows. The manager orders S units from the first supplier and reserves I units from the reliable supplier. Random supply X then arrives from the first supplier. If $X<D$, the inventory manager exercises the option to order $\min\{D-X, I\}$ units from the reliable supplier. If $X < D-I$ the manager orders I units and there is an under stock of $D-I-X$. If $D-I \leq X \leq D$ the manager orders $D-X$ and there is no over or under stock. If $D \leq X$, the inventory manager exercises nothing from the reliable supplier and over stocks by $X-D$.

To understand the manager's actions when uncertainties are bundled, we first analyze the case where the delivery quantity from the first supplier only has recurrent uncertainty (no disruption) represented by a random supply w with cumulative distribution function *G(w)* with a mean *S* (the quantity ordered) and standard deviation σ_w. In the absence of disruption, the expected costs from the perfectly reliable supplier are given by

$$E(TC_{reliable}) = hI + e\int_0^D \min(I, D-w)dG(w).$$

The expected over and under stocking are all attributed to the first supplier and are given by

$$E(TC_{over+under}) = C_u \int_0^{D-I}(D-I-w)dG(w) + C_o\int_D^{\infty}(w-D)dG(w).$$

Given the variable *w* with mean *S*, standard deviation σ_w, and cumulative distribution *G(w)*, define the standardized variable *z* to be

$$z = \frac{w-S}{\sigma_w}.$$

z has the cumulative distribution $G_S(z)$ with mean 0 and standard deviation 1. Given a value *R* of *w,* define

$$(R)^s = \frac{R-S}{\sigma_w}$$

We may denote $(R)^s$ by R^s when there is no ambiguity. Define the standardized loss function

$$\ell(w,(R)^s) = \int_{R^s}^{\infty}(1-G_S(z))dz$$

This yields an expected total cost of (see appendix)

$$E(TC(S,I)) = E(TC_{reliable}) + E(TC_{over+under})$$

$$= (h+e-C_u)I + C_u(D-S) + (C_u-e)\sigma_w\ell\left(w,(D-I)^s\right) + (e+C_o)\sigma_w\ell\left(w,(D)^s\right) \quad (1)$$

The optimal actions by the manager when there is only recurrent uncertainty are obtained in Proposition 1.

Proposition 1: *In the absence of disruption, the order quantity* S^* *from the first supplier is given by*

$$S^* = D - \sigma_w G_S^{-1}\left(\frac{C_o - h}{C_o + e}\right) \tag{2}$$

and the reservation quantity I^* *with the reliable supplier is given by*

$$I^* = Max\left(0, \sigma_w\left(G_S^{-1}\left(\frac{C_o - h}{C_o + e}\right) - G_S^{-1}\left(\frac{h}{C_u - e}\right)\right)\right). \tag{3}$$

Proof: See appendix. $\blacksquare$

The above analysis allows us to understand the manager's actions when she bundles the two risks. Recall that the first supplier has a disruption probability of *p* resulting in a supply of 0 and a recurrent uncertainty represented by a supply *X* with a cumulative distribution function *F(X)* with a mean of *S* (the quantity ordered) and a standard deviation σ_x. Thus, if an order of *S* is placed with the first supplier, the quantity delivered by the first supplier will equal *0* with probability *p* and, with probability *1-p*, will equal *X* which has a cumulative distribution of *F(X)*.

When the manager bundles both sources of uncertainty, let S_1^* be the optimal order quantity with the first supplier, and I_1^* the reservation quantity with the reliable supplier. A manager who bundles the uncertainties expects a random supply *Y* given an order of *S*. The expected value of *Y* is given by

$$E(Y) = (1-p)E(X) = (1-p)S,$$

and its variance is given by

$$Var(Y) = p(1-p)[E(X)]^2 + (1-p)Var(X) = p(1-p)S^2 + (1-p)\sigma_X{}^2. \qquad (4)$$

S_1^* and I_1^* are obtained by replacing w by Y and substituting $S^* = (1-p)S_1^*$, $I^* = I_1^*$, $(1-p)S = E(Y)$, and $\sigma_w = \sigma_Y$ in equations (2) and (3). On bundling, the order quantity S_1^* with the first supplier is given by

$$(1-p)S_1^* = D - \sigma_Y F_s^{-1}\left(\frac{C_o - h}{C_o + e}\right) \qquad (5)$$

and the reservation quantity I_1^* with the reliable supplier is given by

$$I_1^* = Max\left\{0, \sigma_Y\left(F_s^{-1}\left(\frac{C_o - h}{C_o + e}\right) - F_s^{-1}\left(\frac{h}{C_u - e}\right)\right)\right\} \qquad (6)$$

The next step is to evaluate the manager's actions if she decouples the two uncertainties when making her decision. The total cost in this case can again be broken up into two parts: one from contracting with the reliable supplier and one from purchasing from the first supplier. Observe that it is never optimal to reserve more than D units with the reliable supplier, i.e., $D \geq I$. The expected cost for the reliable supplier consists of three components - the cost of reserving quantity I, the cost of purchasing I units and under stocking by D-I units in case of a disruption, and the cost of purchasing the minimum of the reserved quantity I and the shortage D-x in case the supply x is less than the demand D. The expected cost for the reliable supplier is given by

$$E(TC_{reliable}) = hI + p(eI + C_u(D-I)) + (1-p)e\int_0^D \min(I, D-x)dF(x)$$

The expected over and under stocking costs (when supply arrives but leads to over or under stocking) is given by

$$E\left(TC_{over+under}\right)=\left(1-p\right)\left(C_u\int_0^{D-I}(D-I-x)dF(x)+C_o\int_D^{\infty}(x-D)dF(x)\right)$$

The expected total cost on decoupling the two uncertainties is thus given by

$E(TC(S,I)) = E\left(TC_{reliable}\right)+E\left(TC_{over+under}\right)$

$$=hI+p\left(eI+C_u\left(D-I\right)\right)+\left(1-p\right)e\int_0^{D}\min(I,D-x)dF(x)$$

$$+\left(1-p\right)\left(C_u\int_0^{D-I}(D-I-x)dF(w)+C_o\int_D^{\infty}(x-D)dF(x)\right)$$

We thus have

$$E(TC(S,I))=hI+p\left(eI+C_u\left(D-I\right)\right)+\left(1-p\right)\left(\int_0^{D-I}(eI+C_u(D-I-x))\,dF(x)\right)$$

$$+\left(1-p\right)\left(\int_{D-I}^{D}e(D-x)dF(x)+C_o\int_D^{\infty}(x-D)dF(x)\right) \tag{7}$$

Proposition 2 identifies the manager's actions when the uncertainties are decoupled.

Proposition 2: *When the uncertainties are decoupled, the optimal order quantity with the first supplier* S_2^* *is given by*

$$S_2^*=D-\sigma_x F_S^{-1}\left(\frac{\left(1-p\right)\left(C_o+e\right)-h-e+pC_u}{\left(1-p\right)\left(C_o+e\right)}\right), \tag{8}$$

and the optimal reservation quantity from the reliable supplier I_2^* *is given by*

$$I_2^*=\max\left(0,\sigma_x\left(F_S^{-1}\left(\frac{\left(1-p\right)\left(C_o+e\right)-h-e+pC_u}{\left(1-p\right)\left(C_o+e\right)}\right)-F_S^{-1}\left(\frac{h-p\left(C_u-e\right)}{\left(1-p\right)\left(C_u-e\right)}\right)\right)\right). \tag{9}$$

Proof: See appendix. ■

Having identified the manager's actions when she bundles and decouples the risks, we first show that there are instances where bundling the two uncertainties results in the reliable supplier not being used, whereas decoupling the two uncertainties results in the reliable supplier being used.

Proposition 3: *For a positive probability p of disruption for the first supplier, there are values of C_o, C_u, h, and e, such that bundling the two uncertainties results in the reliable supplier not being used, i.e., $I_1^* = 0$, whereas decoupling the two uncertainties results in the reliable supplier being used, i.e., $I_2^* > 0$.*

Proof: From (6) observe that $I_1^* = 0$ if

$$\frac{C_o - h}{C_o + e} \leq \frac{h}{C_u - e} \text{ or}$$

$$h \geq \left(1 - \frac{C_u}{C_u + C_o}\right)(C_u - e) \tag{10}$$

In particular,

$$e = 0 \text{ and } h = C_o \tag{11}$$

result in $I_1^* = 0$.

To obtain I_2^*, we substitute $e = 0$ into (9) to obtain

$$I_2^* = \max\left(0, \sigma_x\left(F_S^{-1}\left(\frac{(1-p)(C_o) - h + pC_u}{(1-p)(C_o)}\right) - F_S^{-1}\left(\frac{h - p(C_u)}{(1-p)(C_u)}\right)\right)\right)$$

Substitute for h from (11) to obtain

$$\frac{(1-p)(C_o)-h+pC_u}{(1-p)(C_o)}-\frac{h-p(C_u)}{(1-p)(C_u)}=\frac{p\,C_u^2-C_o^2}{(1-p)C_o(C_o+C_u)}>0 \text{ for } 1>p>0.$$

This implies that $I_2^* > 0$ using (9). Thus, there are situations where bundling the two uncertainties results in no use of the reliable supplier ($I_1^* = 0$) whereas decoupling the uncertainties results in a positive amount reserved from the reliable supplier ($I_2^* > 0$). $\blacksquare$

Proposition 3 is most closely related to the results of Dada et al. [6]. We show that bundling of risks leads to instances where the reliable supplier is not selected even though it should have been. This relates to the examples of the flu vaccine and Ericsson discussed at the beginning of the paper. Bundling of disruption and recurrent risk is a possible explanation for going with fewer suppliers than may be appropriate in each case.

Next we show in Proposition 4 that when uncertainties are bundled, the quantity ordered from the first supplier increases with the probability of disruption.

Proposition 4: *When the uncertainties are bundled, the quantity ordered from the first supplier* S_1^* *is increasing in the disruption probability p for* $0<p<1$.

Proof: From equation (5) observe that $(1-p)S_1^* = D - \sigma_Y F_S^{-1}\left(\frac{C_o-h}{C_o+e}\right)$. Given that $h+e>C_o$, we obtain $\frac{C_o-h}{C_o+e}<\frac{1}{2}$ or, equivalently, $F_S^{-1}\left(\frac{C_o-h}{C_o+e}\right)<0$. Thus, it follows that S_1^* is increasing in the disruption probability p for $0<p<1$. $\blacksquare$

In contrast, when the uncertainties are decoupled, Proposition 5 shows that the quantity ordered from the first supplier decreases as the probability of disruption grows.

Thus, bundling of recurrent and disruption risk leads to an over utilization of the first supplier.

Proposition 5: *When the uncertainties are decoupled, the quantity ordered from the first supplier* S_2^* *decreases as the probability of disruption p increases.*

Proof: From (8) we obtain

$$S_2^* = D - \sigma_X F_S^{-1}\left(1 - \frac{h + e - pC_u}{(1-p)(C_o + e)}\right)$$

To show that S_2^* decreases with an increase in p, we need to show that

$F_S^{-1}\left(1 - \frac{h + e - pC_u}{(1-p)(C_o + e)}\right)$ increases with an increase in p. This is equivalent to showing that $\left(\frac{h + e - pC_u}{(1-p)(C_o + e)}\right)$ decreases with an increase in p, or $\frac{d}{dp}\left(\frac{h + e - pC_u}{(1-p)(C_o + e)}\right) < 0$

This derivative is given by

$$\frac{d}{dp}\left(\frac{h + e - pC_u}{(1-p)(C_o + e)}\right) = \frac{(C_o + e)((h + e - pC_u) - C_u(1-p))}{[(1-p)(C_o + e)]^2} = \frac{(C_o + e)(h + e - C_u)}{[(1-p)(C_o + e)]^2}$$

Observe that the derivative is negative whenever $h + e < C_u$, a condition we have already assumed from (3). The result thus follows. ■

Proposition 5 makes an important point. Even though the reliable supplier is most useful in the event of a disruption, the reliable supplier also serves the role of mitigating recurrent supply uncertainty. Thus, as the supply uncertainty increases because of an increase in disruption probability, it is best for the manager to mitigate more of the recurrent supply risk using the reliable supplier and use less of the first supplier.

Proposition 6: *When the uncertainties are decoupled, for low disruption probability p and h + e* $\geq C_o$*, the quantity ordered from the first supplier* S_2^* *increases as the recurrent supply uncertainty* σ_x *increases*.

Proof: From (8) recall that

$$S_2^* = D - \sigma_X F_S^{-1}\left(1 - \frac{h + e - pC_u}{(1-p)(C_o + e)}\right)$$

Using the fact that $h + e \geq C_o$, we can show that for low values of p,

$$\left(1 - \frac{h + e - pC_u}{(1-p)(C_o + e)}\right) < \frac{1}{2}.$$

Given that x has been assumed to be symmetric about the mean, we thus obtain

$$F_S^{-1}\left(1 - \frac{h + e - pC_u}{(1-p)(C_o + e)}\right) < 0.$$

The result thus follows. ■

Comparing Propositions 5 and 6 we are able to expand on the insights of Dada et al. [6]. They showed that cost takes precedence over reliability when selecting suppliers. Our results focus on the relative use of the two suppliers once both have been selected. We have shown that the impact of cost and reliability on the relative use of the two suppliers is driven by the source of unreliability. By Proposition 6, if the growth in supply uncertainty is driven by a growth in recurrent uncertainty, using more of the low cost (but unreliable) supplier is a good mitigation strategy. In contrast, if growth in supply uncertainty is driven by a growth in disruption probability, Proposition 5 shows that using more of the reliable supplier and less of the cheaper but unreliable supplier is optimal.

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Insert Figures 1, 2 and 3 Here. Proposed captions:

Figure 1: Optimal Excess Order from First supplier on Bundling and Decoupling

Figure 2: Change in Optimal Excess Order from First Supplier as Disruption Probability and Recurrent Uncertainty Grows

Figure 3: Change in Optimal Reservation Quantity from Reliable Supplier as Disruption Probability and Recurrent Uncertainty Grows

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Numerical experiments confirm all the theoretical conclusions drawn in this section. In all numerical experiments we use D=100, C_o = 10, C_u = 15, e = 8, and h = 2.8 and assume the supply distribution to be normal. Figure 1 shows the change in $S_1^* - 100$, the excess order size from the first (cheaper but less reliable) supplier when risks are bundled, and $S_2^* - 100$, the excess order size from the first supplier when risks are decoupled, as a function of the disruption probability p. In this chart the supply distribution has σ = 15. Observe that when risks are bundled, increasing the disruption probability increases the excess order size ($S_1^* - 100$) from the first supplier. In contrast, when risks are decoupled, increasing the disruption probability decreases the excess order size ($S_2^* - 100$) from the first supplier.

Figure 2 looks at the case where risks are decoupled and shows the impact of changing the recurrent uncertainty σ and the disruption probability p on $S_2^* - 100$, the excess order size from the first supplier. For the upper chart we fix the recurrent uncertainty σ = 15 and vary the disruption probability p from 0.00 to 0.16. In the lower chart we fix the disruption probability p = 0.04 and vary the recurrent uncertainty from σ = 15 to σ = 31. Figure 2 shows that as the probability of disruption increases, the excess quantity ordered from the first supplier ($S_2^* - 100$) should be decreased. In contrast, as the recurrent supply uncertainty increases the excess quantity ordered from the first supplier ($S_2^* - 100$) should be increased.

Figure 3 looks at the case where risks are decoupled and shows the impact of changing the recurrent uncertainty σ and the disruption probability p on I_2^*, the reservation quantity from the reliable supplier. For the upper chart we fix the recurrent uncertainty σ = 15 and vary the disruption probability p from 0.00 to 0.16. In the lower chart we fix the disruption probability p = 0.04 and vary the recurrent uncertainty from σ = 15 to σ = 31. Figure 3 shows that the reservation quantity with the reliable supplier increases with both the disruption probability and the recurrent uncertainty. The disruption probability, however, seems to have a much greater impact on the reservation quantity than the recurrent uncertainty. As the disruption probability grows from 0 to 0.16, the reservation quantity grows from 0 to 6.39. In contrast, as the recurrent uncertainty grows from 0 to 31, the reservation quantity only grows from 0 to 2.80.

To compare the relative use of the first supplier and the reliable supplier to mitigate supply risk, consider the ratio $\left(S_2^* - D\right)/I_2^*$. For the data used in Figures 2 and 3, as the disruption probability increases from 0.02 to 0.16 the ratio $\left(S_2^* - D\right)/I_2^*$ decreases from 5.74 to 0.37. Thus, as the disruption probability increases, more of the supply risk is mitigated by the reliable supplier. In contrast, the ratio $\left(S_2^* - D\right)/I_2^*$ stays constant at 2.53

as the standard deviation of recurrent supply increases from 15 to 31. The first supplier continues to play the dominant role to mitigate recurrent supply uncertainty.

3. CONCLUSION

Dada et al. [6] have shown that cost dominates reliability when selecting suppliers. In this paper we expand on their insights by focusing on the relative use of the two suppliers once both have been selected. We show the importance of recognizing and decoupling disruption and recurrent supply risk when planning mitigation strategies in a supply chain. The managerial implications of our results are as follows:

1. Bundling of disruption and recurrent supply uncertainty results in an over (under) utilization of the low cost (reliable) supplier. The extent of over (under) utilization increases as the probability of disruption grows.
2. Growth in supply risk from increased disruption probability is best mitigated by increased use of the reliable (though more expensive) supplier and decreased use of the cheaper but less reliable supplier. Growth in supply risk from increased recurrent uncertainty, however, is better served by increased use of the cheaper, though less reliable, supplier.

APPENDIX

Derivation of Equation (1): The Expected Total Cost in the Two Supplier Case

$$E(TC(S,I)) = E(TC_{reliable}) + E(TC_{over+under})$$

$$= hI + e\int_0^D \min(I, D-w)dG(w) + C_u \int_0^{D-I}(D-I-w)dG(w) + C_o\int_D^\infty (w-D)dG(w)$$

$$= hI + eI\int_0^{D-I} dG(w) + e\int_{D-I}^{D}(D-w)dG(w) + C_u\int_0^{D-I}(D-I-w)dG(w) + C_o\int_D^\infty (w-D)dG(w)$$

$$= hI + (e-C_u)I\int_0^{D-I} dG(w) + e\int_{D-I}^{D}(D-w)dG(w) + C_u\int_0^{D-I}(D-w)dG(w) + C_o\int_D^\infty (w-D)dG(w)$$

$$= hI + (e-C_u)I\int_0^{D-I} dG(w) + e\int_0^{D}(D-w)dG(w) - e\int_0^{D-I}(D-w)dG(w)$$

$$+ C_u\int_0^{D-I}(D-w)dG(w) + C_o\int_D^\infty (w-D)dG(w)$$

$$= hI + (e-C_u)I\int_0^{D-I} dG(w) + e\int_0^{D}(D-w)dG(w) + (e-C_u)\int_0^{D-I}(w-D)dG(w) + C_o\int_D^\infty (w-D)dG(w)$$

$$= hI + (e-C_u)\int_0^{D-I}(w-(D-I))dG(w) + e\int_0^{D}(D-w)dG(w) + C_o\int_D^\infty (w-D)dG(w)$$

$$= hI + (e-C_u)\int_0^{\infty}(w-(D-I))dG(w) + (C_u-e)\int_{D-I}^{\infty}(w-(D-I))dG(w) + e\int_0^{\infty}(D-w)dG(w)$$

$$- e\int_D^{\infty}(D-w)dG(w) + C_o\int_D^\infty (w-D)dG(w)$$

$$= hI + (e-C_u)I\int_0^{\infty} dG(w) - e\int_0^{\infty}(D-w)dG(w) + C_u\int_0^{\infty}(D-w)dG(w) + (C_u-e)\int_{D-I}^{\infty}(w-(D-I))dG(w)$$

$$+ e\int_0^{\infty}(D-w)dG(w) + (e+C_o)\int_D^\infty (w-D)dG(w)$$

$$= (h+e-C_u)I + C_u(D-S) + (C_u-e)\int_{D-I}^{\infty}(w-(D-I))dG(w) + (e+C_o)\int_D^\infty (w-D)dG(w)$$

Observe that

$$\int_{D}^{\infty}(w-D)dG(w)=\sigma_w \ell\left(w,D^s\right) \text{ and } \int_{D-I}^{\infty}(w-(D-I))dG(w)= \sigma_w \ell\left(w,(D-I)^s\right)$$

We thus have

$$E(TC(S,I))=(h+e-C_u)I+C_u(D-S)+(C_u-e)\sigma_w \ell\left(w,(D-I)^s\right)+(e+C_o)\sigma_w \ell\left(w,D^s\right)$$

$\blacksquare$

Proof of Proposition 1

The proof is provided in the following three steps. Recall that *S* is the expected supply (which is also the quantity ordered) and *I* is the quantity reserved with the reliable supplier.

(a) ***The loss function is convex in S.***

The standardized loss function, $\ell\left(w,D^s\right)=\int_{D^s}^{\infty}(1-G_s(z))dz$, where $z=\frac{(w-S)}{\sigma_w}$ and $D^s=\frac{(D-S)}{\sigma_w}$, is a convex function of *S*.

Proof: Observe that

$$\frac{\partial}{\partial S}\ell\left(w,D^s\right)=\frac{1}{\sigma_w}\left(1-G_s\left(\frac{D-S}{\sigma_w}\right)\right)$$
$$\frac{\partial^2}{\partial S^2}\ell\left(w,D^s\right)=\frac{1}{\sigma_w^2}g_s\left(\frac{D-S}{\sigma_w}\right)\geq 0$$

$\square$

We can similarly prove that the loss function, $\ell\left(w,(D-I)^s\right)=\int_{(D-I)^s}^{\infty}(1-G_s(z))dz$, where $(D-I)^s=\frac{(D-S-I)}{\sigma_w}$, is convex in *S* and *I*.

(b) ***The cost function is convex in S and I.***

Proof: From (1) recall that

$$E(TC(S,I))) = (h+e-C_u)I + C_u(D-S) + (C_u-e)\sigma_w \ell\left(w,(D-I)^s\right) + (e+C_o)\sigma_w \ell\left(w,(D)^s\right)$$

Observe that

$$\begin{aligned}
&\frac{\partial}{\partial I}E(TC(S,I)) = (h+e-C_u) + (C_u-e)\sigma_w \frac{\partial}{\partial I}\ell\left(w,(D-I)^s\right) \\
&\frac{\partial^2}{\partial I^2}E(TC(S,I)) = (C_u-e)\sigma_w \frac{\partial^2}{\partial I^2}\ \ell\left(w,(D-I)^s\right) \geq 0
\end{aligned} \tag{A1}$$

The convexity of *E(TC(S,I))* with respect to *I* follows from the fact $\ell\left(w,(D-I)^s\right)$ is a convex function of *I* as shown earlier and the assumption that $C_u \geq e$.

With regards to *S* observe that

$$\begin{aligned}
&\frac{\partial}{\partial S}E(TC(S,I)) = (-C_u) + (C_u-e)\sigma_w \frac{\partial}{\partial S}\ell\left(w,(D-I)^s\right) + (e+C_o)\sigma_w \frac{\partial}{\partial S}\ell\left(w,(D)^s\right) \\
&\frac{\partial^2}{\partial S^2}E(TC(S,I)) = (C_u-e)\sigma_w \frac{\partial}{\partial S^2}^2\ \ell\left(w,(D-I)^s\right) + (e+C_o)\sigma_w \frac{\partial^2}{\partial S^2}\ell\left(w,(D)^s\right) \geq 0
\end{aligned} \tag{A2}$$

The convexity of *E(TC(S,I)))* with respect to *S* follows if we assume that $C_u > e$, and from the fact that $\ell\left(w,(D-I)^s\right)$ and $\ell\left(w,(D)^s\right)$ are convex functions of *S* as shown earlier.

(c) *The optimal order quantity S^* and reservation quantity I^* are given by*

$$S^* = D - \sigma_w G_s^{-1}\left(\frac{C_o-h}{e+C_o}\right) \text{ and } I^* = Max\left(0, \sigma_w\left(G_s^{-1}\left(\frac{C_o-h}{C_o+e}\right) - G_s^{-1}\left(\frac{h}{C_u-e}\right)\right)\right).$$

Proof: Observe that

$$\begin{aligned}
&\frac{\partial}{\partial I}E(TC(S,I)) = (h+e-C_u) + (C_u-e)\sigma_w \frac{\partial}{\partial I}\ell\left(w,(D-I)^s\right) \\
&\frac{\partial}{\partial S}E(TC(S,I)) = -C_u + (e+C_o)\sigma_w \frac{\partial}{\partial S}\ell\left(w,D^s\right) + (C_u-e)\sigma_w \frac{\partial}{\partial S}\ell\left(w,(D-I)^s\right)
\end{aligned} \tag{A3}$$

From the definition of the standardized loss function observe that

$$\frac{\partial}{\partial S}\ell\left(w,(D-I)^s\right) = \frac{\partial}{\partial I}\ell\left(w,(D-I)^s\right) = \frac{1}{\sigma_w}\left[1 - G_s\left(\frac{D-I-S}{\sigma_w}\right)\right] \tag{A4}$$

$$\frac{\partial}{\partial T}\ell\left(w, D^s\right) = \frac{1}{\sigma_w}\left[1 - G_s\left(\frac{D - S}{\sigma_w}\right)\right].$$

Using (A3) and (A4), we obtain

$$\frac{\partial}{\partial T}E\left(TC(T, I)\right) = -C_u + \left(e + C_o\right)\sigma_w\frac{\partial}{\partial T}\ell\left(w, D^s\right) + \left(C_u - e\right)\sigma_w\frac{\partial}{\partial I}\ell\left(w, (D - I)^s\right)$$

Substituting from (A3) we obtain

$$\frac{\partial}{\partial T}E\left(TC(T, I)\right) = -C_u + \left(e + C_o\right)\sigma_w\frac{\partial}{\partial T}\ell\left(w, D^s\right) + \frac{\partial}{\partial I}E\left(TC(T, I)\right) - \left(h + e - C_u\right)$$

Given that $\frac{\partial}{\partial I}E\left(TC(T, I)\right) = 0$ at optimality (the expected total cost is convex with respect to *I*), we obtain

$$\frac{\partial}{\partial S}E\left(TC(S, I)\right) = -(h + e) + \left(e + C_o\right)\sigma_w\frac{\partial}{\partial S}\ell\left(w, D^s\right) = -(h + e) + \left(e + C_o\right)\left(1 - G_S\left(\frac{D - S}{\sigma_w}\right)\right)$$

Setting $\frac{\partial}{\partial S}E\left(TC(S, I)\right) = 0$, we obtain

$$S^* = D - \sigma_w G_s^{-1}\left(\frac{C_o - h}{e + C_o}\right)$$

I^* is obtained by setting $\frac{\partial}{\partial I}E\left(TC\left(S^*, I^*\right)\right) = 0$, which gives

$$(h + e - C_u) + (C_u - e)\left(1 - G_s\left(\frac{D - I - S^*}{\sigma_w}\right)\right) = 0$$

This implies

$$1 - G_s\left(\frac{D - I^* - S^*}{\sigma_w}\right) = \frac{h + e - C_u}{e - C_u}.$$

Since $I^* \geq 0$, we obtain

$$I^* = Max\left(0, D - S^* - \sigma_w G_s^{-1}\left(\frac{h}{C_u - e}\right)\right) = Max\left(0, \sigma_w\left(G_s^{-1}\left(\frac{C_o - h}{C_o + e}\right) - G_s^{-1}\left(\frac{h}{C_u - e}\right)\right)\right).$$

□

Proof of Proposition 2

$$S_2^* = D - \sigma_x F_s^{-1}\left(\frac{(1-p)(C_o + e) - h - e + pC_u}{(1-p)(C_o + e)}\right) \text{ and}$$

$$I_2^* = \sigma_x\left(F_s^{-1}\left(\frac{(1-p)(C_o + e) - h - e + pC_u}{(1-p)(C_o + e)}\right) - F_s^{-1}\left(\frac{h - p(C_u - e)}{(1-p)(C_u - e)}\right)\right)$$

Proof:

The expected costs can be written as

$$E(TC(S,I)) = hI + p\left(eI + C_u\,(D - I)\right) + (1-p)\left(\int_0^{D-I} (eI + C_u\,(D - I - x))\, dF(x)\right)$$

$$+ (1-p)\left(\int_{D-I}^{D} e(D - x)dF(x) + C_o \int_D^{\infty} (x - D)dF(x)\right)$$

$$= hI + p\left(eI + C_u\,(D - I)\right) + (1-p)\left(\int_0^{D-I} (eI + C_u\,(D - I - x))\, dF(x)\right)$$

$$+ (1-p)\left(e\int_0^{D} (D - x)dF(x) - e\int_0^{D-I} (D - x)dF(x) + C_o \int_D^{\infty} (x - D)dF(x)\right)$$

$$= hI + p\left(eI + C_u\,(D - I)\right) + (1-p)\left(\int_0^{D-I} C_u\,(D - I - x)\, dF(x) - \int_0^{D-I} e(D - I - x)dF(x)\right)$$

$$+ (1-p)\left(e\int_0^{\infty} (D - x)dF(x) - e\int_D^{\infty} (D - x)dF(x) + C_o \int_D^{\infty} (x - D)dF(x)\right)$$

$$= hI + p\left(eI + C_u\,(D - I)\right) + e(1-p)(D - S)$$

$$+(1-p)\left((C_u-e)\int_0^{D-I}(D-I-x)\,dF(x)+(C_o+e)\int_D^{\infty}(x-D)dF(x)\right)$$

$$=hI+p\big(eI+C_u(D-I)\big)+e(1-p)(D-S)$$

$$+(1-p)\left((C_u-e)\int_0^{\infty}(D-I-x)\,dF(x)+(C_u-e)\int_{D-I}^{\infty}(x-(D-I))dF(x)+(C_o+e)\int_D^{\infty}(x-D)dF(x)\right)$$

$$=hI+p\big(eI+C_u(D-I)\big)+e(1-p)(D-S)+(C_u-e)(1-p)(D-I-S)$$

$$+(1-p)\left((C_u-e)\int_{D-I}^{\infty}(x-(D-I))dF(x)+(C_o+e)\int_D^{\infty}(x-D)dF(w)\right)$$

$$E(TC(S,I))=(h+e-C_u)I+C_uD-(1-p)C_uS$$

$$+(1-p)\big((C_u-e)\sigma_x l(x,(D{-}I)^s)+(C_o+e)\sigma_x l(x,D^s)\big)$$

Given that *E(TC(S,I))* is convex in *S* and *I,* we obtain optimality using the first order conditions. Observe that

$$\frac{\partial E(TC(S,I))}{\partial I}=(h+e-C_u)+(1-p)(C_u-e)\sigma_x\frac{\partial}{\partial I}l(x,(D{-}I)^s)\ \text{ and}$$

$$\frac{\partial E(TC(S,I))}{\partial S}=-(1-p)C_u+(1-p)(C_u-e)\sigma_x\frac{\partial}{\partial S}l(x,(D{-}I)^s)+(1-p)(C_o+e)\sigma_x\frac{\partial}{\partial S}l(x,D^s)$$

Given that

$\frac{\partial}{\partial I}l(x,(D{-}I)^s)=\frac{\partial}{\partial T}l(x,(D{-}I)^s)$, we have

$$\frac{\partial E(TC(S,I))}{\partial T}=-(1-p)C_u+\frac{\partial E(TC(S,I))}{\partial I}-(h+e-C_u)+(1-p)(C_o+e)\sigma_x\frac{\partial}{\partial T}l(x,D^s)$$

Using the fact that $\frac{\partial E(TC(S,I))}{\partial I}=0$ at optimality, we have

$$\frac{\partial E(TC(S,I))}{\partial T}=-(1-p)C_u-(h+e-C_u)+(1-p)(C_o+e)\sigma_x\frac{\partial}{\partial T}l(x,D^s)$$

Using the fact that $\frac{\partial}{\partial T}\ell\left(x, D^s\right) = \frac{1}{\sigma_x}\left(1 - F_s\left(\frac{D-S}{\sigma_x}\right)\right)$ and setting $\frac{\partial E(TC(S,I))}{\partial T}$ to be 0,

we obtain

$$(1-p)(C_o+e)\left(1-F_s\left(\frac{D-S_2^*}{\sigma_x}\right)\right) = h+e-pC_u$$

Thus,

$$S_2^* = D - \sigma_x F_s^{-1}\left(\frac{(1-p)(C_o+e)-h-e+pC_u}{(1-p)(C_o+e)}\right)$$

I_2^* is obtained by setting $\frac{\partial}{\partial I}E\left(TC\left(S_2^*, I_2^*\right)\right) = 0$, which gives

$$(h+e-C_u) + (1-p)(C_u-e)\sigma_x \frac{\partial}{\partial I} l(x,(D-I)^s) = 0$$

Substituting

$$\frac{\partial}{\partial I} l(x,(D-I)^s) = \frac{1}{\sigma_x}\left(1 - F_s\left(\frac{D-S-I}{\sigma_x}\right)\right)$$

we obtain

$$h - p(C_u-e) = (1-p)(C_u-e)F_s\left(\frac{D-S_2^*-I_2^*}{\sigma_x}\right)$$

Thus

$$I_2^* = D - S_2^* - \sigma_x F_s^{-1}\left(\frac{h-p(C_u-e)}{(1-p)(C_u-e)}\right)$$

Substituting for $D - S_2^*$, we obtain

$$I_2^* = \sigma_x F_s^{-1}\left(\frac{(1-p)(C_o+e)-h-e+pC_u}{(1-p)(C_o+e)}\right) - \sigma_x F_s^{-1}\left(\frac{h-p(C_u-e)}{(1-p)(C_u-e)}\right)$$

Given that I_2^* must be non-negative, the result follows. ∎

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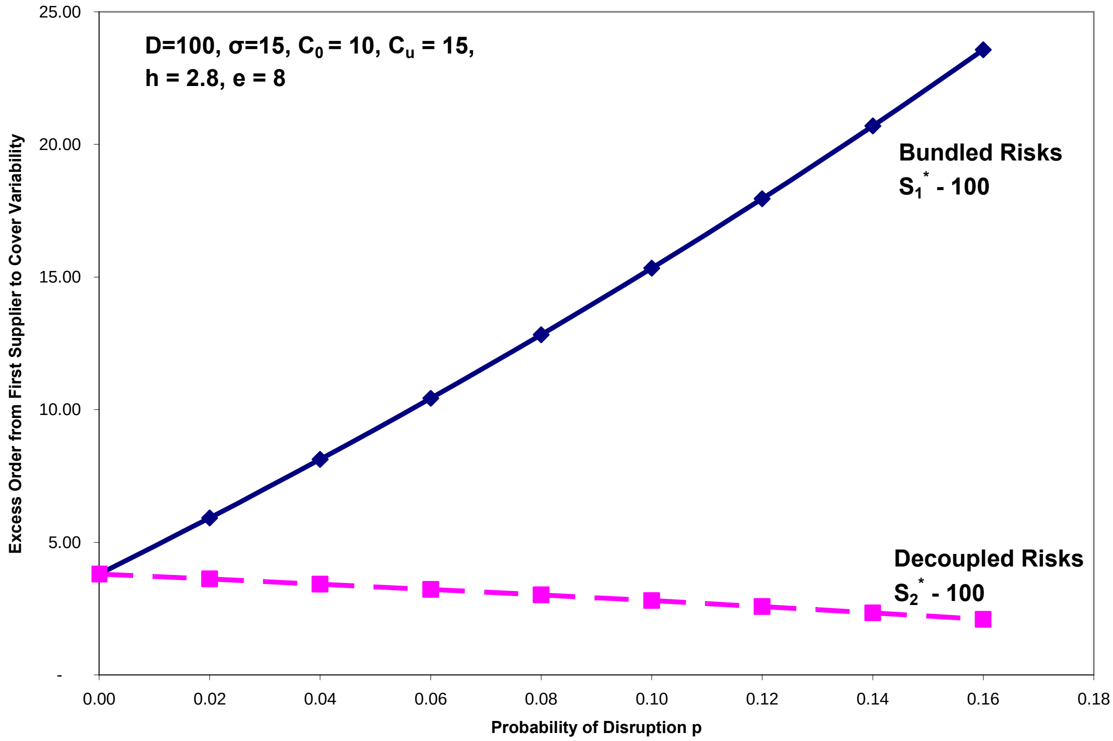
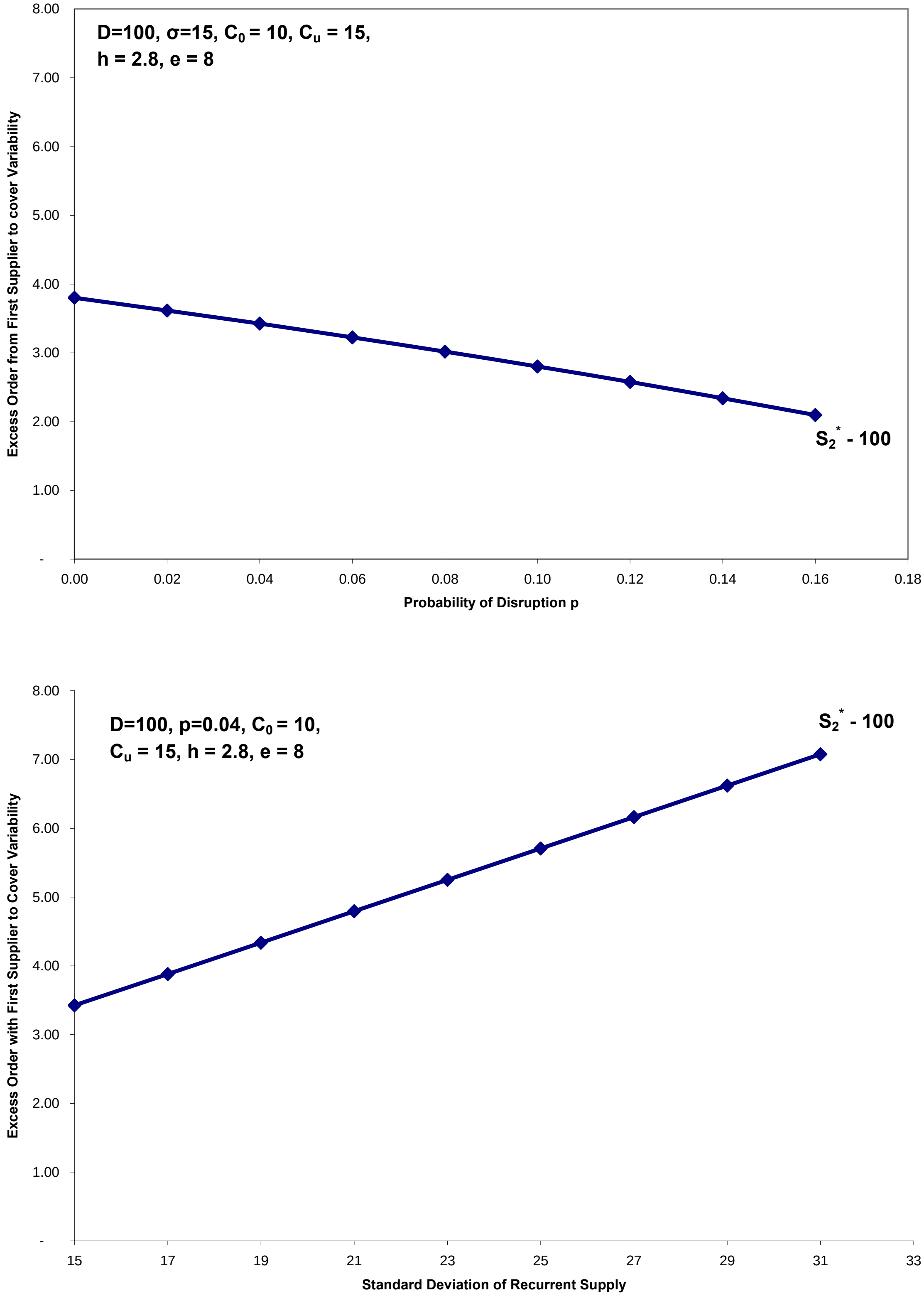
Period	Delivered Amount	Sorted by size
1	83	0
2	94	0
3	108	0
4	0	81
5	114	83
6	89	87
7	0	89
8	92	92
9	109	93
10	118	94
11	81	102
12	116	103
13	0	108
14	87	109
15	103	109
16	109	109
17	93	111
18	102	114
19	111	116
20	109	118
Mean	86	101
St. Dev.	38.60	11.87

Table 1



D=100, σ=15, C0 = 10, Cu = 15,
h = 2.8, e = 8
Bundled Risks
S1* - 100
Decoupled Risks
S2* - 100
Excess Order from First Supplier to Cover Variability
Probability of Disruption p
25.00
20.00
15.00
10.00
5.00
-
0.00
0.02
0.04
0.06
0.08
0.10
0.12
0.14
0.16
0.18

Figure 1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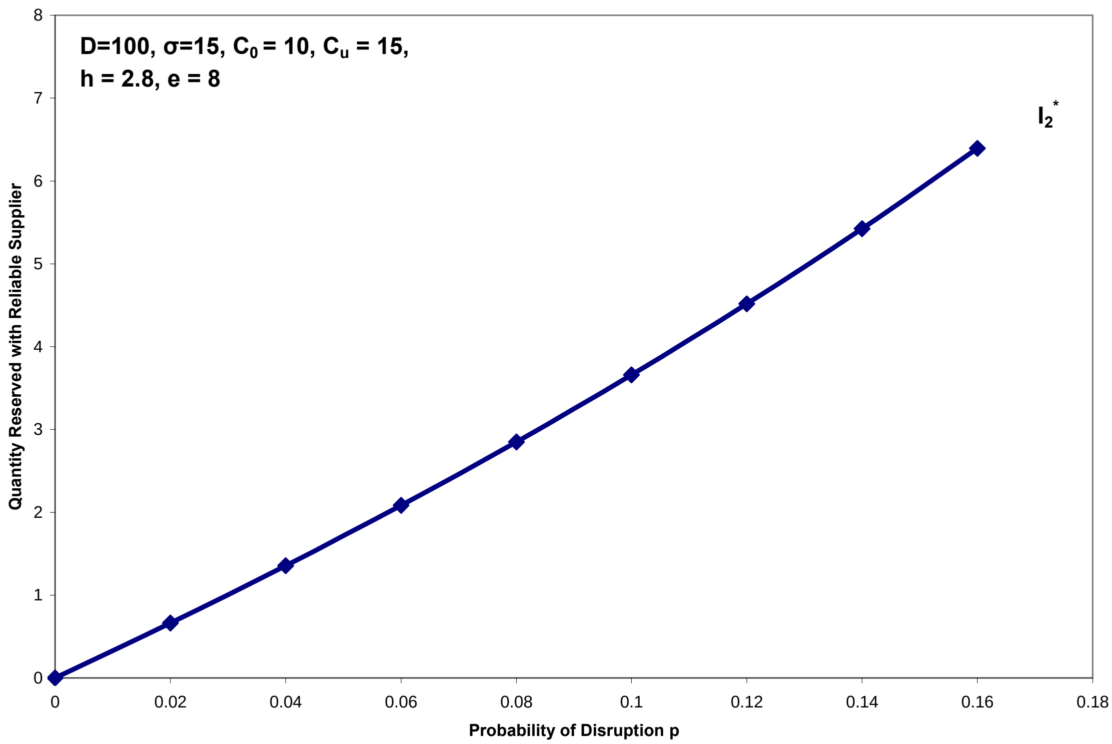
h = 2.8, e = 8
S2* - 100
Excess Order from First Supplier to cover Variability
Probability of Disruption p
8.00
7.00
6.00
5.00
4.00
3.00
2.00
1.00
-
0.00
0.02
0.04
0.06
0.08
0.10
0.12
0.14
0.16
0.18
D=100, p=0.04, C0 = 10,
Cu = 15, h = 2.8, e = 8
S2* - 100
Excess Order with First Supplier to Cover Variability
Standard Deviation of Recurrent Supply
8.00
7.00
6.00
5.00
4.00
3.00
2.00
1.00
-
15
17
19
21
23
25
27
29
31
33

Figure 2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h = 2.8, e = 8
I2*
Quantity Reserved with Reliable Supplier
Probability of Disruption p
0
1
2
3
4
5
6
7
8
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16
0.18

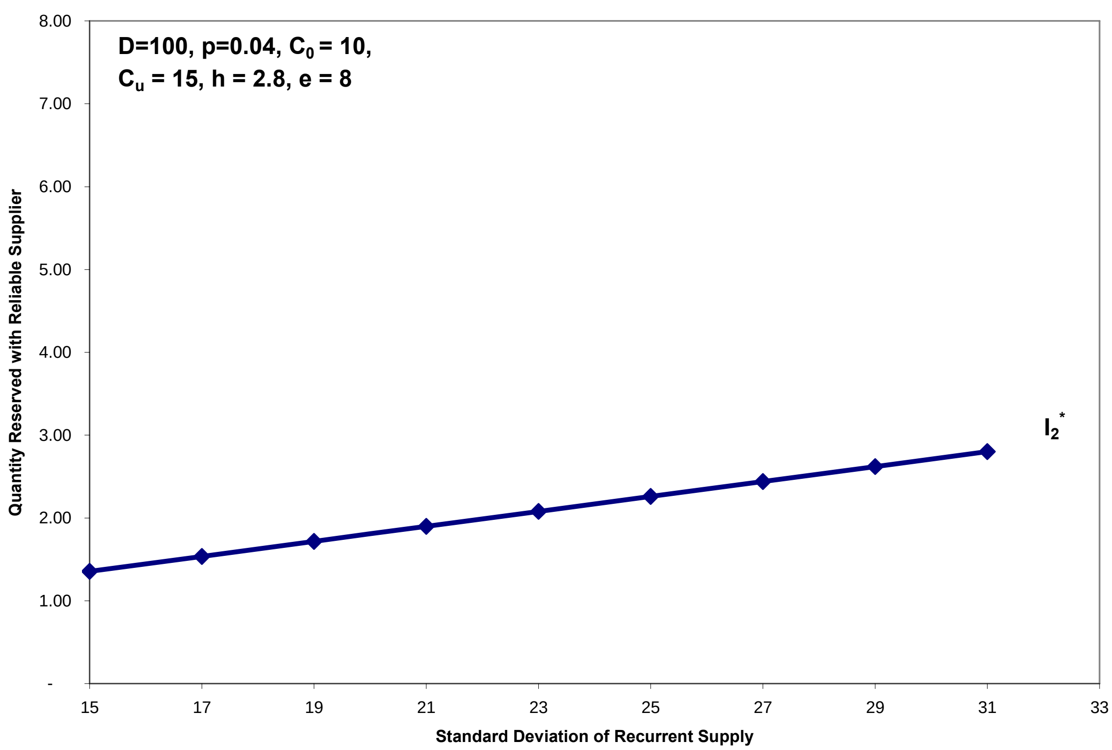
D=100, p=0.04, C0 = 10,
Cu = 15, h = 2.8, e = 8
I2*
Quantity Reserved with Reliable Supplier
Standard Deviation of Recurrent Supply
-
1.00
2.00
3.00
4.00
5.00
6.00
7.00
8.00
15
17
19
21
23
25
27
29
31
33

Figure 3